MA125 Exam 3 Version 1

Name:

1) Find two positive numbers x and y such that $xy = 1$ and the sum

$$x^2 + y^2$$

is as small as possible.

2) The curves $f(x) = x^2$ and $g(x) = \sin x$ intersect somewhere between 0.5 and 1.0. If we are using Newton's method to find the x-coordinate of the point of intersection with a starting value of $x_0 = 1$, what is the value of x_2?

3) Find the absolute maximum and minimum of the function

$$f(x) = x^3 - 6x^2 + 9x + 1 \quad \text{on} \quad [2, 4]$$

4) Find the coordinates of the point on the line

$$y = 4 - x$$

that is closest to $(1, 0)$.

5) Find

$$\lim_{x \to 0} \frac{1 - \cos x}{x^2}$$

6) If

$$f(x) = 3x^2 + 2x + 5$$

Find all numbers c that satisfy the conclusion of the Mean Value Theorem on $[-1, 1]$.

7a) If $f(x) = x^2 - 4x + 1$, which (if any) of the following conclusions can we draw from the Mean Value Theorem?

a) $f'(c) = 5$ for some $c \in (1, 2)$
b) $f'(c) = 5$ for some $c \in (0, 2)$
c) $f'(c) = 5$ for some $c \in (2, 3)$
d) $f'(c) = 5$ for some $c \in (-1, 1)$
e) None of the above

7b) If $f(x) = 2x^3 - 4x^2 - 10x + 12$, which (if any) of the following conclusions can we draw from Rolle's Theorem?

a) $f'(c) = 0$ for some $c \in (0, 2)$
b) $f'(c) = 0$ for some $c \in (1, 4)$
c) $f'(c) = 0$ for some $c \in (2, 3)$
d) $f'(c) = 0$ for some $c \in (-2, 1)$
e) None of the above

7c) If $f'(x) = 0$ for every $x \in (-1, 4)$, which of the following conclusions can be drawn?

a) $f(2) \cdot f(3) > 0$
b) $f(1) = 0$
c) $f(3) - f(0) > 0$
d) $(f(1) - f(2))(f(1) + f(2)) = 0$ if $f(1) \neq 0$
e) None of the above

7d) If $f'(x) = g'(x)$ for $x \in (-3, 3)$, which of the following conclusions can be drawn?

a) The line tangent to f at $x = 1$ intersects the line tangent to g at $x = 2$
b) Secant lines to the graph of f and g drawn with $x_1 = 1$ and $x_2 = 2$ are parallel
c) The function $h(x) = f(x) - g(x)$ has a value of zero on $[-1, 1]$
d) $f(0) = g(0)$
e) None of the above

7e) If f has a local minimum at $x = 0$, what does Fermat's theorem say about $f'(0)$?

a) $f'(0) = 0$
b) if $f'(0)$ exists then $f'(0) = 0$
c) $f'(0) > 0$
d) $f'(0) < 0$
e) Either $f'(0) = 0$ or $f'(0)$ does not exist

Suppose

$$f(x) = 4x^3 + 3x^2 - 6x + 1$$

8a) Which of the following lists contains **all** of the intervals on which f is increasing?

a) $(-\infty, -1)$,$(1/2, \infty)$
b) $(-\infty, 1/2)$
c) $(-\infty, -1), (1, \infty)$
d) $(-1, 1/2)$
e) $(-\infty, \infty)$

8b) Which of the following lists contains **all** of the intervals on which f is decreasing?

a) $(-1, 1/2)$
b) $(-\infty, \infty)$
c) $(-\infty, 1/2)$
d) $(-\infty, -1), (1, \infty)$
e) $(-\infty, -1)$,$(1/2, \infty)$

8c) Which of the following lists contains **all** of the intervals on which f is concave up?

a) $(-1/4, \infty)$
b) $(-1/4, \infty)$
c) $(-\infty, 1/4), (1, \infty)$
d) $(-1/4, 1)$
e) $(-\infty, -1/4)$

8d) Which of the following lists contains **all** of the intervals on which f is concave down?

a) $(-1/4, 1)$
b) $(-1/4, \infty)$
c) $(-\infty, -1/4)$
d) $(-1/4, \infty)$
e) $(-\infty, 1/4), (1, \infty)$

8e) Which of the following lists contains **all** of the intervals on which f' is increasing?

a) $(-\infty, 1/4), (1, \infty)$
b) $(-1/4, 1)$
c) $(-1/4, \infty)$
d) $(-\infty, -1/4)$
e) $(-1/4, \infty)$

9) Find a function f that satisfies:

$$f'(t) = 2t - 3\sin t\ , \quad f(0) = 3$$

10) Find the limit:

$$\lim_{x\to\infty} x^3 e^{-x}$$