MA125 Exam 1 Version 1

Name:

Show all work! No credit will be given for an answer unless I can tell how you got it.

1) Use the **definition of the derivative as a limit of a difference quotient** to find the derivative of the following function:

$$f(x) = \frac{1}{\sqrt{x+1}}$$

(*Do not* use any formulas for a derivative such as of a power of x or others)

2) Suppose $f(x) = 3x - 1$. Use the precise definition of a limit to show that

$$\lim_{x \to 2} f(x) = 5$$

That is, assuming some $\epsilon > 0$ is given, find a $\delta > 0$ such that

$$\text{If} \quad 0 < |x - a| < \delta \quad \text{then} \quad |f(x) - L| < \epsilon$$

3) An object is fired vertically into the air with an initial velocity of $120 ft/sec$. The height above the ground after t seconds is $f(t)$ where

$$f(t) = 120t - 16t^2$$

a)What is the average velocity of the object from $t = 0$ to $t = 3$?

b)What is the average velocity of the object from $t = 3$ to $t = 6$?

c)What is the instantaneous velocity of the object at $t = 4$?

4) Suppose $c \geq 1$ is a constant. Find the following limit, if it exists, or explain why it does not exist.

$$\lim_{x \to \infty} \left(\frac{1}{x + \sin x + c} \right)$$

5 What (if anything) does the Intermediate Value Theorem say about

$$f(x) = \frac{x^2 - 1}{x - 1}$$

assuming a value of $1/2$ for some x in the interval $[0, 2]$? (Be sure to explain your answer in terms of the statement of the theorem and its assumptions).

6) Find the equation of the line tangent to the graph of

$$f(x) = \frac{1}{\sqrt{x+1}}$$

at $x = 0$. (you can make use of your answer to problem 1) in your solution)

7) For what values of x is the function

$$\frac{\sqrt{x^2-1}}{\sqrt{4-x^2}}$$

continuous?

8 A function $f(x)$ is defined piecewise by the following rule of assignment, where b is a positive constant:

$$f(x) = \begin{cases} \frac{x^2-4x}{x+1} & when \quad x \leq 0 \\ 2x-1 & when \quad 0 < x \leq 1 \\ -x^2+4x-2 & when \quad x > 1 \end{cases}$$

Which of the following statements are true and which are false?

T F f is continuous at $x = -2$

T F $\lim_{x\to -1} f(x)$ exists

T F $\lim_{x\to 0^-} f(x)$ exists

T F $\lim_{x\to 0^+} f(x)$ exists

T F $\lim_{x\to 0} f(x)$ exists

T F $\lim_{x\to -1^+} f(x) = -\infty$

T F $\lim_{x\to -1^-} f(x) = -\infty$

T F $f(x)$ is continuous from the right at $x = 0$

T F $f(x)$ is continuous from the left at $x = 0$

T F $f(x)$ is continuous at $x = 0$

T F $f(x)$ is continuous at $x = -1$

T F $\lim_{x\to 1} f(x)$ exists

T F $f(x)$ is continuous at $x = 1$

T F $f(x)$ is continuous from the right at $x = 1$

T F $\lim_{x\to \infty} f(x)$ is a real number

T F $f(x)$ is differentiable at $x = 1$