MA125 Exam 2 Version 1

Name:

1) A clay pot is removed from a kiln at a temperature of $1830°F$ and placed on a rack in a room where the temperature is $68°F$. After 20 minutes, the pot has cooled to $1040°F$. What is the temperature of the pot 45 minutes after it is removed from the kiln?

2) Find the equation of the line tangent to the graph of

$$y = \cosh(\ln x) + 2 \quad \text{at} \quad x = 1$$

3) In pharmacology the **half-life** of a drug refers to the time it takes for the concentration of a drug in the body to be reduced by one half. The half-life of the drug Ibuprofen is 1.9 hours. If a patient takes a dose that produces a concentration of 160mg/l, how long will it be before the concentration reaches 30mg/l? (assume the concentration reaches 160mg/l immediately when the drug is taken).

4) A particle moves along the branch of the hyperbola $x^2 - y^2 = 1$ in the first quadrant ($x \geq 0, y \geq 0$). The x-coordinate of the particle at time t is given by

$$x(t) = 2t^2$$

How fast is the distance from the particle to the origin changing when $x = 4$?

5) Two barges leave from a port at the same time. One sails east at 6mi/hr while the other sails northeast (i.e., at a 45 degree angle to the path of the first barge) at 5mi/hr. How fast is the distance between the two barges increasing one hour after their departure?

6) The height of a cylinder is shrinking at a rate of $5cm/min$ while the radius in growing at a rate of $8cm/min$. How fast is the volume of the cylinder changing when the height is $15cm$ and the radius is $10cm$?

7) The energy E of a particle that obeys the Pauli Exclusion Principle has the Fermi-Dirac probability density

$$f_E = \frac{1}{\exp\left(\frac{E-\mu}{kT}\right) + 1}$$

where μ is the chemical potential, k is the Boltzman constant, and T is the temperature. Find the rate of change of f_E with respect to T (assume μ and E are constant).

8) Find the second derivative of

$$x^3 + xy + y^3 = 1$$

9) Two species of sqirrel (red and grey) are in competition in a certain ecosystem. Let R represent the number of red squirrels and G the number of grey squirrels. In this ecosystem, the Lotka-Volterra model for the size of the two populations is:

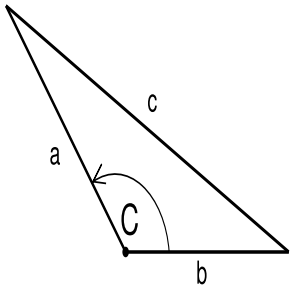
$$\begin{aligned} \frac{dR}{dt} &= R\left(\frac{1000 - R - 3G}{1000}\right) \\ \frac{dG}{dt} &= G\left(\frac{300 - G}{300}\right) \end{aligned}$$

The populations are in equilibrium when both rates of change are zero. Find all population sizes (R, G) for which the populations are in equilibrium.

10) The base of a triangle is increasing at a rate of 3cm/min, while the altitude is increasing at a rate of 2cm/min. How fast is the area of the triangle changing when the area is 100cm^2 and the base is $10cm$?

Formulas:

Volume of a sphere	$V = \frac{4}{3}\pi r^3$
Volume of a cylinder	$V = \pi r^2 h$
Volume of a cone	$V = \frac{1}{3}\pi r^2 h$
Surface area of a sphere	$A = 4\pi r^2$
Hyperbolic functions	$\sinh x = \frac{e^x - e^{-x}}{2}$ $\cosh x = \frac{e^x + e^{-x}}{2}$
Pythagorean theorem	$c^2 = a^2 + b^2$
Law of cosines	$c^2 = a^2 + b^2 - 2ab\cos C$



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MA125 Exam 2 Version 2

Name:

1) Find the second derivative of

$$x^3 - xy + y^3 = 1$$

2) Two species of sqirrel (red and grey) are in competition in a certain ecosystem. Let R represent the number of red squirrels and G the number of grey squirrels. In this ecosystem, the Lotka-Volterra model for the size of the two populations is:

$$\begin{aligned} \frac{dR}{dt} &= R\left(\frac{1000 - R - 2G}{1000}\right) \\ \frac{dG}{dt} &= G\left(\frac{250 - G}{250}\right) \end{aligned}$$

The populations are in equilibrium when both rates of change are zero. Find all population sizes (R, G) for which the populations are in equilibrium.

3) Two barges leave from a port at the same time. One sails east at 5mi/hr while the other sails northeast (i.e., at a 45 degree angle to the path of the first barge) at 4mi/hr. How fast is the distance between the two barges increasing one hour after their departure?

4) A particle moves along the hyperbola $x^2 - y^2 = 1$ in the first quadrant ($x \geq 0, y \geq 0$). The x-coordinate of the particle at time t is given by

$$x(t) = 3t^2$$

How fast is the distance from the particle to the origin changing when $x = 2$?

5) In pharmacology the **half-life** of a drug refers to the time it takes for the concentration of a drug in the body to be reduced by one half. The half-life of the drug Ibuprofen is 1.8 hours. If a patient takes a dose that produces a concentration of 180mg/l, how long will it be before the concentration reaches 30mg/l? (assume the concentration reaches 180mg/l immediately when the drug is taken).

6) The base of a triangle is increasing at a rate of 3cm/min, while the altitude is increasing at a rate of 2cm/min. How fast is the area of the triangle changing when the area is 196cm^2 and the base is $10cm$?

7) The energy E of a particle that does not obey the Pauli Exclusion Principle has the Bose-Einstein probability density

$$f_E = \frac{1}{\exp\left(\frac{E-\mu}{kT}\right) - 1}$$

where μ is the chemical potential, k is the Boltzman constant, and T is the temperature. Find the rate of change of f_E with respect to T (assume μ and E are constant).

8) Find the equation of the line tangent to the graph of

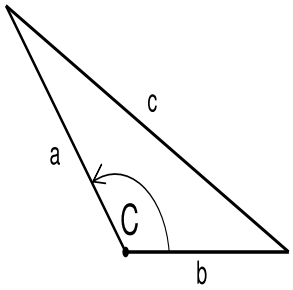
$$y = \cosh(\ln x) - 2 \quad \text{at} \quad x = 1$$

9) A clay pot is removed from a kiln at a temperature of $1830°F$ and placed on a rack in a room where the temperature is $68°F$. After 25 minutes, the pot has cooled to $1040°F$. What is the temperature of the pot 45 minutes after it is removed from the kiln?

10) The height of a cylinder is shrinking at a rate of $8cm/min$ while the radius in growing at a rate of $5cm/min$. How fast is the volume of the cylinder changing when the height is $15cm$ and the radius is $10cm$?

Formulas:

Volume of a sphere	$V = \frac{4}{3}\pi r^3$
Volume of a cylinder	$V = \pi r^2 h$
Volume of a cone	$V = \frac{1}{3}\pi r^2 h$
Surface area of a sphere	$A = 4\pi r^2$
Hyperbolic functions	$\sinh x = \frac{e^x - e^{-x}}{2}$ $\cosh x = \frac{e^x + e^{-x}}{2}$
Pythagorean theorem	$c^2 = a^2 + b^2$
Law of cosines	$c^2 = a^2 + b^2 - 2ab\cos C$



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