MA125 Exam 1 Version 1

Name:

Show all work! No credit will be given for an answer unless I can tell how you got it.

1) Use the **definition of the derivative as a limit of a difference quotient** to find the derivative of the following function:

$$f(x) = \frac{1}{x^2 - 2}$$

(*Do not* use any formulas for a derivative such as of a power of x or others)

2) Suppose $f(x) = 2x - 4$. Use the precise definition of a limit to show that

$$\lim_{x \to 3} f(x) = 2$$

That is, assuming some $\epsilon > 0$ is given, find a $\delta > 0$ such that

$$\text{If} \quad 0 < |x - a| < \delta \quad \text{then} \quad |f(x) - L| < \epsilon$$

3) An object is fired from a height of $5,000$ feet on a vertical trajectory towards the ground. The initial velocity is $-80 ft/sec$. The height above the ground after t seconds is $f(t)$ where

$$f(t) = 5000 - 80t - 16t^2$$

a)What is the average velocity of the object from $t = 1$ to $t = 2$?

b)What is the instantaneous velocity of the object at $t = 3$?

c)What is the instantaneous acceleration of the object at $t = 2.8$?

4) Suppose c is a positive constant. Find the following limit, if it exists, or explain why it does not exist.

$$\lim_{x\to\infty}\left(\sqrt{x^2-c}-x\right)\cos x$$

5 Prove that there exists a number c in the interval $[0, \infty)$ such that

$$f(x) = e^x - \ln(x + 1)$$

assumes the value 10 when $x = c$ (you do not have to find c, just prove that it exists). If you use any theorems, be sure to show that the assumptions they require are true.

6) Find the equation of the line tangent to the graph of

$$f(x) = \frac{1}{x} - \sqrt{x}$$

at $x = 1$.

7) Find a number a such that

$$\lim_{x \to 2} \frac{x^2 - ax + a}{x - 2}$$

exists, or show that there is no such number.

8 A function $f(x)$ is defined piecewise by the following rule of assignment, where b is a positive constant:

$$f(x) = \begin{cases} \frac{2x^2+x-1}{x+1} & when \quad x < -1 \\ x - 2 & when \quad -1 \leq x \leq 0 \\ 2 + \ln x & when \quad 0 < x < 1 \\ \frac{6x^2+3x+1}{x^2+4} & when \quad x \geq 1 \end{cases}$$

Which of the following statements are true and which are false? (Give some justification, however brief, for /textbfeach answer)

$T \quad F \quad \lim_{x \to -1^-} f(x)$ exists

$T \quad F \quad \lim_{x \to -1^+} f(x)$ exists

$T \quad F \quad \lim_{x \to -1} f(x)$ exists

$T \quad F \quad f(x)$ is continuous from the left at $x = -1$

$T \quad F \quad f(x)$ is continuous at $x = -1$

$T \quad F \quad f(x)$ is differentiable at $x = -1$

$T \quad F \quad \lim_{x \to 0^+} f(x) = -\infty$

$T \quad F \quad \lim_{x \to 0^-} f(x)$ exists

$T \quad F \quad f(x)$ is continuous from the right at $x = 0$

$T \quad F \quad f(x)$ is continuous from the left at $x = 0$

$T \quad F \quad f(x)$ is continuous at $x = 0$

$T \quad F \quad f(x)$ is continuous at $x = 1/2$

$T \quad F \quad \lim_{x \to 1} f(x)$ exists

$T \quad F \quad f(x)$ is continuous at $x = 1$

$T \quad F \quad f(x)$ is continuous from the right at $x = 1$

$T \quad F \quad \lim_{x \to \infty} f(x)$ exists

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MA125 Exam 1 Version 2

Name:

Show all work! No credit will be given for an answer unless I can tell how you got it.

1) Use the **definition of the derivative as a limit of a difference quotient** to find the derivative of the following function:

$$f(x) = \frac{1}{x^2 + 2}$$

(*Do not* use any formulas for a derivative such as of a power of x or others)

2 Prove that there exists a number c in the interval $[1, \infty)$ such that

$$f(x) = e^{(x-1)} - \ln(x)$$

assumes the value 5 when $x = c$ (you do not have to find c, just prove that it exists). If you use any theorems, be sure to show that the assumptions they require are true.

3) Find the equation of the line tangent to the graph of

$$f(x) = \frac{1}{x} + \sqrt{x}$$

at $x = 1$.

4) Find a number a such that

$$\lim_{x \to 2} \frac{x^2 - ax + a}{x - 2}$$

exists, or show that there is no such number.

5) Suppose $f(x) = 2x - 1$. Use the precise definition of a limit to show that

$$\lim_{x \to 3} f(x) = 5$$

That is, assuming some $\epsilon > 0$ is given, find a $\delta > 0$ such that

$$\text{If} \quad 0 < |x - a| < \delta \quad \text{then} \quad |f(x) - L| < \epsilon$$

6) An object is fired from a height of $4,000$ feet on a vertical trajectory towards the ground. The initial velocity is $-80ft/sec$. The height above the ground after t seconds is $f(t)$ where

$$f(t) = 4000 - 80t - 16t^2$$

a)What is the average velocity of the object from $t = 1$ to $t = 2$?

b)What is the instantaneous velocity of the object at $t = 3$?

c)What is the instantaneous acceleration of the object at $t = 2.8$?

7) Suppose c is a positive constant. Find the following limit, if it exists, or explain why it does not exist.

$$\lim_{x\to\infty}\left(\sqrt{x^2+c}-x\right)\cos x$$

8 A function $f(x)$ is defined piecewise by the following rule of assignment, where b is a positive constant:

$$f(x) = \begin{cases} \frac{2x^2+x-1}{x+1} & when \quad x < -1 \\ x - 2 & when \quad -1 \leq x \leq 0 \\ 2 + \ln x & when \quad 0 < x < 1 \\ \frac{6x^2+3x+1}{x^2+4} & when \quad x \geq 1 \end{cases}$$

Which of the following statements are true and which are false? (Give some justification, however brief, for /textbfeach answer)

$T \quad F \quad \lim_{x \to -1^-} f(x)$ exists

$T \quad F \quad \lim_{x \to -1^+} f(x)$ exists

$T \quad F \quad \lim_{x \to -1} f(x)$ exists

$T \quad F \quad f(x)$ is continuous from the left at $x = -1$

$T \quad F \quad f(x)$ is continuous at $x = -1$

$T \quad F \quad f(x)$ is differentiable at $x = -1$

$T \quad F \quad \lim_{x \to 0^+} f(x) = -\infty$

$T \quad F \quad \lim_{x \to 0^-} f(x)$ exists

$T \quad F \quad f(x)$ is continuous from the right at $x = 0$

$T \quad F \quad f(x)$ is continuous from the left at $x = 0$

$T \quad F \quad f(x)$ is continuous at $x = 0$

$T \quad F \quad f(x)$ is continuous at $x = 1/2$

$T \quad F \quad \lim_{x \to 1} f(x)$ exists

$T \quad F \quad f(x)$ is continuous at $x = 1$

$T \quad F \quad f(x)$ is continuous from the right at $x = 1$

$T \quad F \quad \lim_{x \to \infty} f(x)$ exists

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