

Name:

1) Random variable Y has density function:

$$f(y) = \frac{1}{2}y^2e^{-y}, \quad y \in [0, \infty)$$

Find the density function of the random variable $U = Y^2$.

2) A random vector $Y = \{Y_1, Y_2, \dots, Y_n\}$ has n independent, identically distributed components each with density function

$$f(y_i) = \frac{1}{\beta}e^{-y/\beta}, \quad y \in [0, \infty)$$

a) Find the density function of $U = Y_1 + Y_2 + \dots + Y_n$.

b) Find the mean and variance of U .

3) A random variable Y has density function

$$\begin{cases} f(y) & \text{if } 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the density function and interval of support for the random variable

$$U = aY, \quad a > 0$$

4) A random vector $Y = \{Y_1, Y_2, \dots, Y_n\}$ has n independent, identically distributed components each with a uniform distribution on $[0, 1]$.

a) Find the density function $g_{(k)}(y_k)$ of the k^{th} order statistic, where $1 < k < n$. (Section 6.7)

b) Find the mean and variance of the k^{th} order statistic $Y_{(k)}$. (hint: see if you can recognize the density function in part a) as one that appears in the back cover of the text).

5) Suppose Y_1 has a chi-square distribution with 6 degrees of freedom, and Y_2 has a chi-square distribution with 15 degrees of freedom, and Y_1 and Y_2 are independently distributed.

a) Identify the distribution of the random variable $U = Y_1 + Y_2$.

b) What is the density function of U ?

c) What are the mean and variance of U ?