Name:

1) Random variable X and Y have joint density function:

$$f(x,y) = \begin{cases} a \cdot (x^2y + 2xy^2) & \text{if } \ 0 \le x \le 1, \ 0 \le y \le 2 \\ 0 & \text{elsewhere} \end{cases}$$

a) Find the value of a that makes f a valid joint density function.

b) Find the joint cumulative distribution function $F(x,y)$

c) Find marginal density function of x, $f_x(x)$

d) Find marginal density function of y, $f_y(y)$

d) Find the expected value and variance of X, $E(X)$ and $V(X)$

e) Find the expected value and variance of Y, $E(Y)$ and $V(Y)$

f) Find the covariance of X and Y, $\text{Cov}(X, Y)$

g) Find the conditional density $f_{x|y}(x)$ of X given Y

h) Find the conditional density $f_{y|x}(y)$ of Y given X

2) A random variable Y has density function

$$f(y) = \begin{cases} a \cdot e^{-(2x+y)} & \text{if} \quad x, y \in [0, \infty) \\ 0 & \quad\quad \text{elsewhere} \end{cases}$$

a) Find the value of a that makes f a valid joint density function.

b) Find the joint cumulative distribution function $F(x, y)$

c) Find marginal density function of x, $f_x(x)$

d) Find marginal density function of y, $f_y(y)$

d) Find the expected value and variance of X, $E(X)$ and $V(X)$

e) Find the expected value and variance of Y, $E(Y)$ and $V(Y)$

f) Find the covariance of X and Y, $\text{Cov}(X, Y)$

g) Find the conditional density $f_{x|y}(x)$ of X given Y

h) Find the conditional density $f_{y|x}(y)$ of Y given X

3) A random vector $Y = \{Y_1, Y_2, \ldots, Y_n\}$ consists of n independent, identically distributed random variables Y_i, where each Y_i has a beta distribution,

$$f(y_i) = \left[\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\right] y_i^{\alpha-1}(1-y_i)^{\beta-1}, \quad y_i \in (0,1), \quad i = 1, \ldots, n$$

and α and β are positive constants common to all y_i.

a) Find the joint density function of the random vector Y.

b) Suppose $\overline{y}$ is the mean of the Y_i,

$$\overline{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$$

Use Theorem 5.12 to find the mean and variance of $\overline{y}$. (hint: $a_1 = a_2 = \cdots = a_n = 1/n$)