MA396 In-Class Exercise - Group I

Names:

a) b)
c) d)
e) f)

A *quadratic form* is a construct of the form $x'Ax$ where x is a vector and A is a matrix.

For an arbitrary 2×2 matrix A,

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

the *determinant* of A, denoted by $|A|$, is the scalar quantity $ad - bc$.

The *inverse* of a 2×2 matrix A with $|A| \neq 0$ is:

$$A^{-1} = \frac{1}{|A|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

(If $|A| = 0$, A does not have an inverse). If X is a vector of n random variables $(X_1, \ldots, X_n)$ with expected value $\mu = (\mu_1, \ldots, \mu_n)$ and variance-covariance matrix V, then X is said to have the *multivariate normal* distribution if the joint density function of X is:

$$f_X(x_1, \ldots, x_n) = \frac{1}{\left(\sqrt{2\pi}\right)^n \sqrt{|V|}} \exp\left(-\frac{1}{2}(x-\mu)'V^{-1}(x-\mu)\right)$$

where $x = (x_1, \ldots, x_n)$, provided $|V| \neq 0$. The notation $X \sim N(\mu, V)$ is used to indicate that X has a multivariate normal distribution with mean vector μ and variance-covariance matrix V.

In this situation, the (multivariate) moment-generating function $M_X(t)$ of X is:

$$M_X(t) = \exp\left(t'\mu + \frac{1}{2}t'Vt\right)$$

where $t = t_1, \ldots, t_n)$ is a vector of parameters.

1) Find A^{-1} and the value of the quadratic form $u'Au$ if

$$u = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \text{and} \quad A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

2) Suppose $X = (X_1, X_2) \sim N(\mu, A)$ where $\mu = (1, 0)$ is the vector of means. What is the multivariate density function of X?

3) What is the moment generating function of X?

4) If b is a vector of constants, the mean of $b'X$ is $b'\mu$, and the variance of $b'X$ is $b'Vb$. Find the mean and variance of $Y = X_1 - X_2$.

MA396 In-Class Exercise - Group II

Names:

a)	b)
c)	d)
e)	f)

A *quadratic form* is a construct of the form $x'Ax$ where x is a vector and A is a matrix.

For an arbitrary 2×2 matrix A,

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

the *determinant* of A, denoted by $|A|$, is the scalar quantity $ad - bc$.

The *inverse* of a 2×2 matrix A with $|A| \neq 0$ is:

$$A^{-1} = \frac{1}{|A|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

(If $|A| = 0$, A does not have an inverse). If X is a vector of n random variables $(X_1, \ldots, X_n)$ with expected value $\mu = (\mu_1, \ldots, \mu_n)$ and variance-covariance matrix V, then X is said to have the *multivariate normal* distribution if the joint density function of X is:

$$f_X(x_1, \ldots, x_n) = \frac{1}{\left(\sqrt{2\pi}\right)^n \sqrt{|V|}} \exp\left(-\frac{1}{2}(x-\mu)'V^{-1}(x-\mu)\right)$$

where $x = (x_1, \ldots, x_n)$, provided $|V| \neq 0$. The notation $X \sim N(\mu, V)$ is used to indicate that X has a multivariate normal distribution with mean vector μ and variance-covariance matrix V.

In this situation, the (multivariate) moment-generating function $M_X(t)$ of X is:

$$M_X(t) = \exp\left(t'\mu + \frac{1}{2}t'Vt\right)$$

where $t = t_1, \ldots, t_n)$ is a vector of parameters.

1) Find A^{-1} and the value of the quadratic form $u'Au$ if

$$u = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \text{and} \quad A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

2) Suppose $X = (X_1, X_2) \sim N(\mu, A)$ where $\mu = (0, 1)$ is the vector of means. What is the multivariate density function of X?

3) What is the moment generating function of X?

4) If b is a vector of constants, the mean of $b'X$ is $b'\mu$, and the variance of $b'X$ is $b'Vb$. Find the mean and variance of $Y = X_1 - X_2$.

MA396 In-Class Exercise - Group III

Names:

a)	b)
c)	d)
e)	f)

A *quadratic form* is a construct of the form $x'Ax$ where x is a vector and A is a matrix.

For an arbitrary 2×2 matrix A,

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

the *determinant* of A, denoted by $|A|$, is the scalar quantity $ad - bc$.

The *inverse* of a 2×2 matrix A with $|A| \neq 0$ is:

$$A^{-1} = \frac{1}{|A|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

(If $|A| = 0$, A does not have an inverse). If X is a vector of n random variables $(X_1, \ldots, X_n)$ with expected value $\mu = (\mu_1, \ldots, \mu_n)$ and variance-covariance matrix V, then X is said to have the *multivariate normal* distribution if the joint density function of X is:

$$f_X(x_1, \ldots, x_n) = \frac{1}{\left(\sqrt{2\pi}\right)^n \sqrt{|V|}} \exp\left(-\frac{1}{2}(x-\mu)'V^{-1}(x-\mu)\right)$$

where $x = (x_1, \ldots, x_n)$, provided $|V| \neq 0$. The notation $X \sim N(\mu, V)$ is used to indicate that X has a multivariate normal distribution with mean vector μ and variance-covariance matrix V.

In this situation, the (multivariate) moment-generating function $M_X(t)$ of X is:

$$M_X(t) = \exp\left(t'\mu + \frac{1}{2}t'Vt\right)$$

where $t = t_1, \ldots, t_n)$ is a vector of parameters.

1) Find A^{-1} and the value of the quadratic form $u'Au$ if

$$u = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \text{and} \quad A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

2) Suppose $X = (X_1, X_2) \sim N(\mu, A)$ where $\mu = (1, 1)$ is the vector of means. What is the multivariate density function of X?

3) What is the moment generating function of X?

4) If b is a vector of constants, the mean of $b'X$ is $b'\mu$, and the variance of $b'X$ is $b'Vb$. Find the mean and variance of $Y = X_1 - X_2$.

MA396 In-Class Exercise - Group IV

Names:

a)	b)
c)	d)
e)	f)

A *quadratic form* is a construct of the form $x'Ax$ where x is a vector and A is a matrix.

For an arbitrary 2×2 matrix A,

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

the *determinant* of A, denoted by $|A|$, is the scalar quantity $ad - bc$.

The *inverse* of a 2×2 matrix A with $|A| \neq 0$ is:

$$A^{-1} = \frac{1}{|A|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

(If $|A| = 0$, A does not have an inverse). If X is a vector of n random variables $(X_1, \ldots, X_n)$ with expected value $\mu = (\mu_1, \ldots, \mu_n)$ and variance-covariance matrix V, then X is said to have the *multivariate normal* distribution if the joint density function of X is:

$$f_X(x_1, \ldots, x_n) = \frac{1}{\left(\sqrt{2\pi}\right)^n \sqrt{|V|}} \exp\left(-\frac{1}{2}(x-\mu)'V^{-1}(x-\mu)\right)$$

where $x = (x_1, \ldots, x_n)$, provided $|V| \neq 0$. The notation $X \sim N(\mu, V)$ is used to indicate that X has a multivariate normal distribution with mean vector μ and variance-covariance matrix V.

In this situation, the (multivariate) moment-generating function $M_X(t)$ of X is:

$$M_X(t) = \exp\left(t'\mu + \frac{1}{2}t'Vt\right)$$

where $t = t_1, \ldots, t_n)$ is a vector of parameters.

1) Find A^{-1} and the value of the quadratic form $u'Au$ if

$$u = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \text{and} \quad A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

2) Suppose $X = (X_1, X_2) \sim N(\mu, A)$ where $\mu = (0, 0)$ is the vector of means. What is the multivariate density function of X?

3) What is the moment generating function of X?

4) If b is a vector of constants, the mean of $b'X$ is $b'\mu$, and the variance of $b'X$ is $b'Vb$. Find the mean and variance of $Y = X_1 + X_2$.

MA396 In-Class Exercise - Group V

Names:

a)	b)
c)	d)
e)	f)

A *quadratic form* is a construct of the form $x'Ax$ where x is a vector and A is a matrix.

For an arbitrary 2×2 matrix A,

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

the *determinant* of A, denoted by $|A|$, is the scalar quantity $ad - bc$.

The *inverse* of a 2×2 matrix A with $|A| \neq 0$ is:

$$A^{-1} = \frac{1}{|A|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

(If $|A| = 0$, A does not have an inverse). If X is a vector of n random variables $(X_1, \ldots, X_n)$ with expected value $\mu = (\mu_1, \ldots, \mu_n)$ and variance-covariance matrix V, then X is said to have the *multivariate normal* distribution if the joint density function of X is:

$$f_X(x_1, \ldots, x_n) = \frac{1}{\left(\sqrt{2\pi}\right)^n \sqrt{|V|}} \exp\left(-\frac{1}{2}(x-\mu)'V^{-1}(x-\mu)\right)$$

where $x = (x_1, \ldots, x_n)$, provided $|V| \neq 0$. The notation $X \sim N(\mu, V)$ is used to indicate that X has a multivariate normal distribution with mean vector μ and variance-covariance matrix V.

In this situation, the (multivariate) moment-generating function $M_X(t)$ of X is:

$$M_X(t) = \exp\left(t'\mu + \frac{1}{2}t'Vt\right)$$

where $t = t_1, \ldots, t_n)$ is a vector of parameters.

1) Find A^{-1} and the value of the quadratic form $u'Au$ if

$$u = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \text{and} \quad A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$$

2) Suppose $X = (X_1, X_2) \sim N(\mu, A)$ where $\mu = (1, 1)$ is the vector of means. What is the multivariate density function of X?

3) What is the moment generating function of X?

4) If b is a vector of constants, the mean of $b'X$ is $b'\mu$, and the variance of $b'X$ is $b'Vb$. Find the mean and variance of $Y = X_1 - 2X_2$.

MA396 In-Class Exercise - Group VI

Names:

a)	b)
c)	d)
e)	f)

A *quadratic form* is a construct of the form $x'Ax$ where x is a vector and A is a matrix.

For an arbitrary 2×2 matrix A,

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

the *determinant* of A, denoted by $|A|$, is the scalar quantity $ad - bc$.

The *inverse* of a 2×2 matrix A with $|A| \neq 0$ is:

$$A^{-1} = \frac{1}{|A|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

(If $|A| = 0$, A does not have an inverse). If X is a vector of n random variables $(X_1, \ldots, X_n)$ with expected value $\mu = (\mu_1, \ldots, \mu_n)$ and variance-covariance matrix V, then X is said to have the *multivariate normal* distribution if the joint density function of X is:

$$f_X(x_1, \ldots, x_n) = \frac{1}{\left(\sqrt{2\pi}\right)^n \sqrt{|V|}} \exp\left(-\frac{1}{2}(x-\mu)'V^{-1}(x-\mu)\right)$$

where $x = (x_1, \ldots, x_n)$, provided $|V| \neq 0$. The notation $X \sim N(\mu, V)$ is used to indicate that X has a multivariate normal distribution with mean vector μ and variance-covariance matrix V.

In this situation, the (multivariate) moment-generating function $M_X(t)$ of X is:

$$M_X(t) = \exp\left(t'\mu + \frac{1}{2}t'Vt\right)$$

where $t = t_1, \ldots, t_n)$ is a vector of parameters.

1) Find A^{-1} and the value of the quadratic form $u'Au$ if

$$u = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \text{and} \quad A = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$$

2) Suppose $X = (X_1, X_2) \sim N(\mu, A)$ where $\mu = (0, 1)$ is the vector of means. What is the multivariate density function of X?

3) What is the moment generating function of X?

4) If b is a vector of constants, the mean of $b'X$ is $b'\mu$, and the variance of $b'X$ is $b'Vb$. Find the mean and variance of $Y = 2X_1 - X_2$.