Name:

1) A random variable Y has density function

$$f(y) = \begin{cases} a \cdot (2 - y) & \text{if} \quad -2 \le y \le 2 \\ 0 & \text{elsewhere} \end{cases}$$

a) Find the value of a that makes f a valid density function.

b) Find the cumulative distribution function $F(y)$

c) Find the expected value $E(Y)$

d) Find the variance $V(Y)$

2) A random variable Y has density function

$$f(y) = \begin{cases} k \cdot e^{-y} & \text{if } y \in [1, \infty) \\ 0 & \text{elsewhere} \end{cases}$$

a) Find the value of k that makes f a valid density function.

b) Find the cumulative distribution function $F(y)$

c) Find the expected value $E(Y)$

d) Find the variance $V(Y)$

e) Find the moment-generating function $m(t)$

3) A random variable Y has density function

$$f(y) = \begin{cases} k & \text{if } y \in [1, 5] \\ 0 & \text{elsewhere} \end{cases}$$

a) Find the value of k that makes f a valid density function.

b) Find the cumulative distribution function $F(y)$

c) Find the expected value $E(Y)$

d) Find the variance $V(Y)$

4) A random variable Y has cumulative distribution function

$$F(y) = \begin{cases} y^{3/2} - 1 & \text{if } y \in [0, 1] \\ 0 & \text{elsewhere} \end{cases}$$

a) Find the density function $f(y)$.

b) Find the expected value $E(Y)$

c) Find the variance $V(Y)$

d) Find the median $\phi_{.5}$

5) Suppose Y has expected value $E(Y) = \mu = 50$ and variance $V(Y) = 16$.

   a) Find an upper bound for the probability that Y takes a value outside the interval $[38, 62]$

b) Find a lower bound for the probability that Y takes a value in the interval $[34, 66]$

c) Find a value d such that the probability that Y takes a value outside the interval $[50 - d, 50 + d]$ is less than or equal to $1/10$.