Name:

1) Suppose $Y_1, \ldots, Y_6$ is a random sample of size 6 from a Weibull distribution

$$f(y) = \frac{m}{\alpha} y^{m-1} e^{-y^m/\alpha}, \quad y > 0$$

Show that if m is known, then

$$\sum_{i=1}^{6} y_i^m \quad \text{is sufficient for } \alpha$$

2) Let $Y_1, \ldots, Y_n$ be a random sample of size n from a geometric distribution. Use the factorization theorem to show that $\overline{Y}$ is sufficient for p.

3) Suppose $Y_1, Y_2, \ldots, Y_n$ is a random sample and each Y_i has density function

$$f(y) = \left(\frac{2y}{\theta}\right) e^{-y^2/\theta} \quad y > 0$$

a) Show that $\sum_{i=1}^{n} y_i^2$ is sufficient for θ.

b) Show that

$$\hat{\theta} = \frac{1}{n}\sum_{i=1}^{n} y_i^2$$

is a MVUE of θ (hint: show that $\hat{\theta}$ is a function of a sufficient statistic and is unbiased)

4) Suppose $Y_1, Y_2, \ldots, Y_n$ is a random sample and each Y_i has density function

$$f(y) = \alpha\beta^{\alpha} y^{-(\alpha+1)} \quad y > 0$$

Show that if β is known then $\prod_{i=1}^{n} y_i$ is sufficient for α.