1)

$\alpha = .05$, $z_{\alpha/2} = 1.96$, which can be obtained with the spreadsheet formula $= NORMSINV(1-\alpha/2)$, so the 95% confidence interval is:

$$0.78 \pm 1.96\sqrt{\frac{(.78)(.22)}{1000}} \quad = \quad 0.78 \pm .026 \quad = \quad (.754, .806)$$

The margin of error is 2.6 percent. Assuming the worst case, which occurs when $p = 0.5$, produces a margin of error of 3.1 percent.

2) Problem 8.64 $\alpha = .02$, $z_{\alpha/2} = 2.326$, so the 98% confidence interval is:

$$(0.77-0.35) \pm 2.326\sqrt{\frac{(.77)(.23)}{150} + \frac{(.35)(.65)}{340}} \quad = \quad 0.42 \pm .10 \quad = \quad (.32, .52)$$

The margin of error is 2.6 percent. Assuming the worst case, which occurs when $p = 0.5$, produces a margin of error of 3.1 percent.

b) No, .60 is outside the confidence interval.

3)

From a spreadsheet or calculator, we determine that:

	Spring	Summer
n	5	4
s^2	98.06	582.26
mean	15.62	72.28

The pooled variance is:

$$s_p^2 = \frac{(4)(98.06) + (3)(582.26)}{7} = 305.57$$

and the 95% confidence interval will use a $t_{\alpha/2}$ with $5+4-2 = 7$ degrees of freedom. Using the table in the text, or the spreadsheet function $= TINV(.05, 7)$, we determine that $t_{\alpha/2} = 2.365$

$$(15.62-72.28) \pm 2.365\sqrt{305.57\left(\frac{1}{5} + \frac{1}{4}\right)} = -56.6 \pm 27.73 = (-84.39, -28.93)$$

We assumed that the two random samples were independently selected from normal populations with equal variances.

4) The pooled variance is:

$$s_p^2 = \frac{(3)(.001) + (4)(.002)}{7} = .0016$$

and the 95% confidence interval will use a $t_{\alpha/2}$ with $4+5-2=7$ degrees of freedom. Using the table in the text, or the spreadsheet function $=TINV(.05,7)$, we determine that $t_{\alpha/2}=2.365$

$$(.22-.17)\pm 2.365\sqrt{.0016\left(\frac{1}{4}+\frac{1}{5}\right)}=.05\pm .063=(-0.013,0.113)$$

5) Problem 8.93

a) If σ is known, use the results for linear combinations,

$$E(t'Y)=t'\mu \quad \text{and} \quad V(t'Y)=t'Vt$$

or Theorem 3.12 together with the fact that the sum of independent normals is also normal to determine that

$$2\overline{X}+\overline{Y}$$

has a normal distribution with:

$$\text{mean}=2\mu_1+\mu_2 \quad \text{and variance}=\sigma^2\left(\frac{4}{n}+\frac{3}{m}\right)$$

so that a 95% confidence interval for $2\mu_1+\mu_2$ is

$$2\overline{x}+\overline{y}\pm 1.96\sigma\sqrt{\frac{4}{n}+\frac{3}{m}}$$

If σ is not known, use the fact that

$$\frac{1}{\sigma^2}\sum_{i=1}^{n}\left(X_i-\overline{X}\right)^2$$

has a Chi-square distribution with $n-1$ degrees of freedom, and similarly

$$\frac{1}{3\sigma^2}\sum_{i=1}^{m}\left(Y_i-\overline{Y}\right)^2$$

has a Chi-square distribution with $m-1$ degrees of freedom, so their sum is a Chi-square variate with $n+m-2$ degrees of freedom. Then use Definition 7.2 with

$$\hat{\sigma}^2=\frac{\sum_{i=1}^{n}\left(X_i-\overline{X}\right)^2+\frac{1}{3}\sum_{i=1}^{m}\left(Y_i-\overline{Y}\right)^2}{n+m-2}$$

to determine that

$$T = \frac{(2\overline{X} + \overline{Y}) - (2\mu_1 + \mu_2)}{\hat{\sigma}^2\sqrt{\frac{4}{m} + \frac{3}{m}}}$$

has a t distribution with $n + m - 2$ degrees of freedom, so the 95% confidence interval is given by

$$2\overline{X} + \overline{Y} \pm t_{.025}\hat{\sigma}\sqrt{\frac{4}{m} + \frac{3}{m}}$$

6) With $n = 20$, the sample variance is $s^2 = 34854.4$. From Section 8.9, modifying to produce a one-sided confidence interval, we get:

$$P\left[\sigma^2 \leq \frac{(n-1)S^2}{\chi^2_{1-\alpha}}\right] = 1 - \alpha$$

with $\chi^2_{.99,19}$ computed as =CHIINV(.99,19) with result 7.6327. Since for positive values square roots preserve order, we can write

$$P\left[\sigma^2 \leq \sqrt{\frac{(n-1)S^2}{\chi^2_{1-\alpha}}}\right] = 1 - \alpha$$

so the upper limit is

$$\sqrt{\frac{(19)(34854.4)}{7.6327}} = 294.55$$

Since 150 hours is below the upper limit, we conclude that it is possible that the true population standard deviation might be 150 hours, or even less.