Name:

1) Suppose $(Y_1, \ldots, Y_5)$ is a random sample of size 5 from a $N(\mu, 1)$ distribution, and that

$$\hat{\mu_1} = Y_1 + Y_3 + Y_5 - Y_2 - Y_4 \quad \text{and} \quad \hat{\mu_2} = \overline{Y} = \frac{1}{5}\sum_{i=1}^{5} Y_i$$

Show that $\hat{\mu_1}$ and $\hat{\mu_2}$ are unbiased and find the efficiency of $\hat{\mu}_1$ relative to $\hat{\mu}_2$.

2) If Y has a a binomial distribution with n trials and probability of success p, show that Y/n is a consistent estimator for p.

3) Suppose $Y_1, Y_2, \ldots, Y_n$ is a random sample and each Y_i has density function

$$f(y) = \begin{cases} \theta y^{\theta-1} & \text{if} \quad 0 \le y \le 1 \\ 0 & \textit{otherwise} \end{cases}$$

Show that $\overline{Y}$ is a consistent estimator for $\theta/(\theta+1)$.

4) Let $Y_1, Y_2, \ldots, Y_n$ be a random sample from a gamma distribution with parameters α and β. Show that $\overline{Y}$ converges in probability to a constant and find that constant.

5) Suppose $Y_1, Y_2, \ldots, Y_n$ is a random sample from the probability density function

$$f(y) = \begin{cases} \frac{2}{y^2} & \text{if} \quad y \geq 2 \\ 0 & \quad \textit{otherwise} \end{cases}$$

Determine whether or not the law of large numbers applies to $\overline{Y}$.