MA396 Exam 1 Name:

1) Suppose $Y_1, \ldots, Y_n$ is a random sample from a normal population with mean zero and standard deviation σ.

a) Show that $w = \sum_{i=1}^{n} Y_i^2$ is a sufficient statistic for σ.

b) Show that $u = w/\sigma^2$ has a chi-square distribution and find the associated degrees of freedom.

2) With reference to the statistic u in part b) of problem 1), find the MVUE of σ^2 by finding a function $h(u)$ such that $h(u)$ is an unbiased estimate of σ^2, that is, find $h(u)$ with the property that $E(h(u)) = \sigma^2$.

3) For the random sample described in problem 1), find the method of moments estimate of σ^2

4) For the random sample described in problem 1), find the maximum likelihood estimate of σ^2.

5) Use the statistic from problem 1b) to write a formula for a $100(1-\alpha)$ percent (two-sided) confidence interval for σ^2. Use your formula to find the lower and upper limits when the sample size is 15 and $\alpha = .05$.

6) Suppose $Y_1, Y_2, \dots, Y_n$ is a random sample from an exponential population:

$$f(y) = \frac{1}{\beta} e^{-y/\beta}, \quad y > 0$$

Show that if

$$Y_{(1)} = \min(Y_1, \dots, Y_n)$$

then

$$\hat{\beta} = n \cdot Y_{(1)}$$

is an unbiased estimator for β and find $MSE(\hat{\beta})$

7) As in problem 6), suppose $Y_1, Y_2, \ldots, Y_n$ is a random sample from an exponential population:

$$f(y) = \frac{1}{\beta} e^{-y/\beta}, \quad y > 0$$

Show that

$$\frac{2}{\beta} \sum_{i=1}^{n} Y_i$$

is a pivotal quantity with a chi-square distribution, and use it to derive a 99% confidence interval for β.

8) 150 items are selected from each of two production lines, Line 1 and Line 2. The sample from Line 1 had 27 defectives and the sample from Line 2 had 18 defectives. Find a 95% for the true difference in the proportions of defectives from the two lines.

9) Two drugs were given to patients with high cholesterol levels. The first lowered the cholesterol of 18 patients an average of 17 points with a standard deviation of 5 points, while the second lowered the cholesterol of 20 patients an average of 13 points with a standard deviation of 8 points. Find a 95% confidence interval for the difference in the mean reductions in cholesterol levels. You may assume that the measurements are normally distributed with eqal variances.

10) Suppose $Y_1, \ldots, Y_n$ is a random sample from a population with density function

$$f(y) = 3y^2 \quad 0 < y < 1$$

Show that the sample mean $\overline{Y}$ converges in probability to 3/4.

11) Suppose $Y_1, \ldots, Y_n$ is a random sample from a $N(\mu, \sigma)$ population. Find the maximum likelihood estimate of σ.

12) Suppose $Y_1, \ldots, Y_n$ is a random sample from a Poisson distribution with mean λ. Find the maximum likelihood estimate of λ.