MA396 In-Class Exercise - Group I

**Names:**

a) b)
c) d)
e) f)

**1)** A random variable $Y$ has density function

$$f_Y(y) = \frac{|y|}{25}, \quad y \in [-5, 5]$$

a) Determine the support of $Y$, that is, $S = \{y : f_Y(y) > 0\}$.

b) Show that $f_Y$ is a valid pdf (i.e, that it's nonnegative and its integral over its support is 1). (hint: break the integral into two parts depending on the sign of $y$).

c) Find the expected value of $Y$, $\mathrm{E}(Y)$, if it exists.

d) Show that $\mathrm{Var}(Y) = 12.5$

**2)** A random variable $Y$ has density function

$$f_Y(y) = \frac{1}{y^2}, \quad y \in [1, \infty)$$

a) Determine the support of $Y$, that is, $S = \{y : f_Y(y) > 0\}$.

b) Show that $f_Y$ is a valid pdf (i.e, that it's nonnegative and its integral over its support is 1).

c) Find the expected value of $Y$, $\mathrm{E}(Y)$, if it exists.

d) The value $y_{90}$ with the property that $P(Y \leq y_{90}) = 0.9$ is called the $90^{th}$ *percentile* of $Y$. Show that 10 is the $90^{th}$ percentile of the random variable $Y$ defined above.

MA396 In-Class Exercise - Group II

**Names:**

a) b)
c) d)
e) f)

**1)** A random variable $Y$ has density function

$$f_Y(y) = \frac{3y^2}{2000}, \quad y \in [-10, 10]$$

a) Determine the support of $Y$, that is, $S = \{y : f_Y(y) > 0\}$.

b) Show that $f_Y$ is a valid pdf (i.e, that it's nonnegative and its integral over its support is 1). (hint: break the integral into two parts depending on the sign of $y$).

c) Find the expected value of $Y$, $\mathrm{E}(Y)$, if it exists.

d) Show that $\mathrm{Var}(Y) = 60$

**2)** A random variable $Y$ has density function

$$f_Y(y) = \frac{1}{2}e^{-|y|}, \quad y \in \mathbb{R}$$

a) Determine the support of $Y$, that is, $S = \{y : f_Y(y) > 0\}$.

b) Show that $f_Y$ is a valid pdf (i.e, that it's nonnegative and its integral over its support is 1). (hint: break the integral into two parts depending on the sign of $y$).

c) Find the expected value of $Y$, $\mathrm{E}(Y)$, if it exists.

d) Show that for $a > 0$, $P(-a \leq y \leq a) = 1 - e^{-a}$

MA396 In-Class Exercise - Group III

**Names:**

a) b)
c) d)
e) f)

**1)** A random variable $Y$ has density function

$$f_Y(y) \;=\; \frac{1}{10}, \quad x \in [-5, 5]$$

a) Determine the support of $Y$, that is, $S = \{y \,:\, f_Y(y) > 0\}$.

b) Show that $f_Y$ is a valid pdf (i.e, that it's nonnegative and its integral over its support is 1).

c) Find the expected value of $Y$, $\mathrm{E}(Y)$, if it exists.

d) Show that $\mathrm{Var}(Y) = 25/3$

**2)** A random variable $Y$ has density function

$$f_Y(y) \;=\; \frac{e^y}{e^3 - 1}, \quad y \in [0, 3]$$

a) Determine the support of $Y$, that is, $S = \{y \,:\, f_Y(y) > 0\}$.

b) Show that $f_Y$ is a valid pdf (i.e, that it's nonnegative and its integral over its support is 1).

c) Find the expected value of $Y$, $\mathrm{E}(Y)$, if it exists.

d) Find the moment generating function

$$M_Y(t) \;=\; \mathrm{E}(e^{ty})$$

MA396 In-Class Exercise - Group IV

**Names:**

a) b)
c) d)
e) f)

**1)** A random variable $Y$ has density function

$$f_Y(y) \;=\; \frac{1}{x \cdot \ln(5)}, \quad x \in [1, 5]$$

a) Determine the support of $Y$, that is, $S = \{y \,:\, f_Y(y) > 0\}$.

b) Show that $f_Y$ is a valid pdf (i.e, that it's nonnegative and its integral over its support is 1).

c) Find the expected value of $Y$, $\mathrm{E}(Y)$, if it exists.

d) Show that

$$\mathrm{E}(Y^2) \;=\; \frac{12}{\ln 5}$$

**2)** A random variable $Y$ has density function

$$f_Y(y) \;=\; \cos y, \quad y \in [0, \frac{\pi}{2}]$$

a) Determine the support of $Y$, that is, $S = \{y \,:\, f_Y(y) > 0\}$.

b) Show that $f_Y$ is a valid pdf (i.e, that it's nonnegative and its integral over its support is 1).

c) Find the expected value of $Y$, $\mathrm{E}(Y)$, if it exists.

d) Find the variance of $Y$.

MA396 In-Class Exercise - Group V

**Names:**

a) b)
c) d)
e) f)

**1)** A random variable $Y$ has density function

$$f_Y(y) \;=\; \frac{3+x}{36}, \quad x \in [0,6]$$

a) Determine the support of $Y$, that is, $S = \{y \,:\, f_Y(y) > 0\}$.

b) Show that $f_Y$ is a valid pdf (i.e, that it's nonnegative and its integral over its support is 1).

c) Find the expected value of $Y$, $\mathrm{E}(Y)$, if it exists.

d) Show that

$$\mathrm{E}(Y^2) \;=\; 15$$

**2)** A random variable $Y$ has density function

$$f_Y(y) \;=\; \frac{\cos y}{2}, \quad y \in [-\frac{\pi}{2}, \frac{\pi}{2}]$$

a) Determine the support of $Y$, that is, $S = \{y \,:\, f_Y(y) > 0\}$.

b) Show that $f_Y$ is a valid pdf (i.e, that it's nonnegative and its integral over its support is 1).

c) Find the expected value of $Y$, $\mathrm{E}(Y)$, if it exists.

d) Find the variance of $Y$.

MA396 In-Class Exercise - Group VI

**Names:**

a) b)
c) d)
e) f)

**1)** A random variable $Y$ has density function

$$f_Y(y) = \frac{10-x}{50}, \quad x \in [0, 10]$$

a) Determine the support of $Y$, that is, $S = \{y : f_Y(y) > 0\}$.

b) Show that $f_Y$ is a valid pdf (i.e, that it's nonnegative and its integral over its support is 1).

c) Find the expected value of $Y$, $\mathrm{E}(Y)$, if it exists.

d) Find $\mathrm{Var}(Y)$, if it exists.

**2)** A random variable $Y$ has density function

$$f_Y(y) = \frac{2}{\pi(1+x^2)}, \quad y \in [-1, 1]$$

a) Determine the support of $Y$, that is, $S = \{y : f_Y(y) > 0\}$.

b) Show that $f_Y$ is a valid pdf (i.e, that it's nonnegative and its integral over its support is 1).

c) Find the expected value of $Y$, $\mathrm{E}(Y)$, if it exists.

d) Find the variance of $Y$.