MA396 Assignment 1

Name:

Background: The density function of the standard normal distribution is:

$$f(x) \;=\; \frac{1}{\sqrt{2\pi}}e^{-x^2/2}, \quad x \in (-\infty, \infty)$$

There is a spreadsheet linked to the assignment page that contains the following:

Sheet 1 (Standard Normal Distribution)

- 100 observations X_i from a Uniform density (Cells $A2 : A101$)
- 100 observations z_i from a standard normal distribution (Cells $B2 : B101$)
- An interval for z (Lower limit in Cell $C2$, upper limit in Cell $D2$) (you can change these values)
- 100 indicator variables: 1 if Lower limit $\leq z_i \leq$ Upper limit, else 0
- The sum of the indicator variables (Cell $F2$)
- The proportion of the indicator variables that are 1 (Cell $G2$)
- The theoretical probability that a standard normal variable falls in the interval $[LL, UL]$ (Cell $H2$):

$$P(\text{Lower limit} \leq X \leq \text{Upper limit}) \quad = \quad \int_{LL}^{UL} \frac{1}{\sqrt{2\pi}}e^{-x^2/2}\,dx$$

Sheet 2 (Binomial Distribution)

- Number of trials n for the binomial distribution (Cell $D2$)
- Probability of success p on each trial (Cell $E2$)
- Number of Successes k, $k = 0, 1, \ldots n$ (Cells $A(k+2)$)
- Probability of exactly k successes, $k = 0, 1, \ldots, 10$ (Cell $B(2+k)$)
- Probability of k or fewer successes, $k = 0, 1, \ldots, 10$ (Cell $C(2+k)$)
- 10 uniform random variates for simulating Bernoulli trials (Cells $A15 - A24$)
- 10 simulated Bernoulli trial outcomes (Cells $B15 - B24$)
- Number of successes in 10 simulated Bernoulli trials (Cell $D14$)

The probability of exactly k successes in n trials when the probability of success on each trial is p and $q = 1 - p$ is given by:

$$P(k \text{ successes in } n \text{ trials}) = \binom{n}{k} p^k q^{n-k}, \quad k = 0, 1, \ldots, n$$

Recall that:

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

The probability of k or fewer successes, also known as the *cumulative* probability, is:

$$P(k \text{ or fewer successes in } n \text{ trials}) = \sum_{i=0}^{k} \binom{n}{i} p^i q^{n-i}$$

Use the spreadsheet attached to the assignment page to answer the following questions (Note that changing any cell results in a new sample of 100 observations. Each pair of questions will be answered from a different sample).

1) What is the probability that a single observation from a standard normal population falls in the interval $[-1, 1]$?

2) What proportion of your sample of 100 actually fell in this range?

3) What is the probability that a single observation from a standard normal population falls in the interval $[-2, 2]$?

4) What proportion of your sample of 100 actually fell in this range?

5) What is the probability that a single observation from a standard normal population falls in the interval $[-3, 3]$?

6) What proportion of your sample of 100 actually fell in this range?

7) What is the probability that a single observation from a standard normal population falls in the interval $[-10, 0]$?

8) What proportion of your sample of 100 actually fell in this range?

9) With the lower limit set at -10, experiment with the upper limit to find the value that make the theoretical probability that an observation falls in this interval 0.95. What is this value?

10) Set the low limit equal to the upper limit with a minus sign. Adjust the two values (keeping them equal in absolute value) to find the limits that make the theoretical probability that an observation falls in this interval 0.50. What is this value?

11) What is the probability that 10 tosses of a fair coin results in exactly 5 heads?

12) How many heads were obtained in the 10 simulated tosses (Cell D14)?

13) Suppose a baseball player has a .250 average. What is the probability of the player getting more than 3 hits in 10 at bats? More than 4 hits?

14) How many 'hits' (successes) were obtained in 10 simulated at bats (Cell D14)?

15) A certain drug cures a disease in 75% of the patients suffering from the disease who receive it. If 10 patients are given the drug, what is the probability that 6 or fewer are cured?