MA395 Set Theory Assignment Solution

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Set Theory Assignment

Let

$$S = \{1, 2, 3, 4, 5\}$$

Find three different algebras of sets over S,

 $\mathfrak{F}_1(S)$ $\mathfrak{F}_2(S)$ $\mathfrak{F}_3(S)$

Solution

Two algebras of sets over $S = \{1, 2, 3, 4, 5\}$ were described in the class notes:

$$\begin{array}{rcl}
\mathfrak{Q}_{1} &=& \{\{1,2,3,4,5\}, \emptyset\} \\
\mathfrak{Q}_{2} &=& \{\mathcal{P}(S)\}
\end{array}$$

Solution

Two algebras of sets over $S = \{1, 2, 3, 4, 5\}$ were described in the class notes:

One way to produce a third algebra of sets over S is to extend Q_1 by adding a proper subset of S. Since the collection must be closed under complementation, we also have to add the complement of the new subset.

Suppose we choose to add $\{1,2\}$ to Q_1 . Since the collection of subsets comprising the new algebra of sets must be closed under complimentation, we have to add the compliment of the new set, $\{3,4,5\}$ as well.

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The resulting collection is then

$$Q_3 = \{\{1, 2, 3, 4, 5\}, \{1, 2\}, \{3, 4, 5\}, \emptyset\}$$

You can verify that Q_3 is an algebra of sets over S.

If you add any proper subset of Q_1 and its compliment to Q_1 , the result will be a new algebra of sets over *S*.