Section 3.9: Further Properties of the Mean and Variance

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Expectation of Multivariate Functions

Theorem: (3.9.1 - discrete case) Suppose X and Y are discrete random variables with joint pdf $p_{XY}(x, y)$ and $g(X, Y)$ is a function of X and Y.

Then the expected value of the random variable $g(X, Y)$ is given by

$$E\left[g(X,Y)\right] = \sum_{\text{all } x} \sum_{\text{all } y} g(X,Y) \cdot p_{XY}(x,y)$$

Expectation of Multivariate Functions

Theorem: (3.9.1 - continuous case) Suppose X and Y are continuous random variables with joint pdf $f_{XY}(x, y)$ and $g(X, Y)$ is a continuous function.

Then the expected value of the random variable $g(X, Y)$ is given by

$$E\left[g(X, Y)\right] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x, y) \cdot f_{XY}(x, y)\, dx dy$$

provided that:

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |g(x, y)| \cdot f_{XY}(x, y)\, dx dy \; < \; \infty$$

Expectation of Sums

Theorem: (3.9.2) Suppose X and Y are random variables with finite expected values, and a and b are any real numbers. Then

$$E(aX + bY) \;=\; a \cdot E(X) + b \cdot E(Y)$$

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Corollary: Let $W_1, W_2, \ldots, W_n$ be any set of n random variables, each with finite expectation.

Also, let $a_1, a_2, \ldots, a_n$ be any set of n constants.

Then:

$$E(a_1W_1 + a_2W_2 + \cdots + a_nW_n) \;=\; a_1E(W_1) + a_2E(W_2) + \cdots + a_nE(W_n)$$

Expectation of a Product of Independent Variables

Theorem: (3.9.2) Suppose X and Y are *independent* random variables with finite expected values. Then

$$E(XY) = E(X) \cdot E(Y)$$

Variance of a Sum

Theorem: (3.9.4) Suppose $W_1, W_2, \ldots, W_n$ are independent random variables each with finite variance. Then

$$\text{Var}(W_1 + W_2 + \cdots + W_n) \;=\; \text{Var}(W_1) + \text{Var}(W_2) + \cdots + \text{Var}(W_n)$$

Variance of a Sum

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Corollary: Let $W_1, W_2, \ldots, W_n$ be any set of n random variables, each with finite variance.

Also, let $a_1, a_2, \ldots, a_n$ be any set of n constants.

Then

$$\text{Var}(a_1W_1 + a_2W_2 + \cdots + a_nW_n) = a_1^2\text{Var}(W_1) + a_2^2\text{Var}(W_2) + \cdots + a_n^2\text{Var}(W_n)$$