Larson and Marx Section 3.6

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Definition of Variance

Definition: Suppose a discrete random variable X has probability density function $p_X(k)$.

The **variance** of X, denoed by $Var(X)$ or σ^2 is given by:

$$Var(X) = \sigma^2 \quad = \quad \sum_{\text{all } k} (k-\mu)^2 \cdot p_X(k)$$

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The sum is taken over all values k that the random variable can assume.

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Definition: Suppose a continuous random variable Y has pdf $f_Y(y)$.

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Equivalently, integral may also be taken over the subset of the real line where f_Y has support.

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The variance of a discrete random variable X is **undefined** if

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The Standard Deviation

In applications, the measure of dispersion most commonly used is called the **standard deviation**, and is defined for a discrete random variable X by:

$$\sigma = \sqrt{Var(X)} = \sqrt{\sigma^2} \quad = \quad \sqrt{\sum_{\text{all } k} (k-\mu)^2 \cdot p_X(k)}$$

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For a continuous random variable Y, the standard deviation is defined to be:

$$\sigma = \sqrt{Var(Y)} = \sqrt{\sigma^2} = \quad \sqrt{\int_{-\infty}^{\infty} (y - \mu)^2 \cdot f_Y(y)\, dy}$$

Relationship of Mean and Variance

Theorem: Suppose W is a random variable with mean μ for which

$$E(W^2) \quad \text{is finite}$$

then

$$Var(W) = \sigma^2 = \quad E(W^2) - [E(W)]^2 \quad = \quad E(W^2) - \mu^2$$

Higher Moments

Definition: Suppose W is a random variable with pdf $f_W(w)$. For any positive integer r, $p_Y(y)$ and $g(Y)$ is a function of Y.

The r^{th} **moment about the origin** μ_r is given by:

$$\mu_r = E(W^r)$$

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The r^{th} **moment about the mean** μ_r' is given by:

$$\mu_r' = E[(W-\mu)^r]$$

provided the expectation on the right hand side is finite.