Larson and Marx Section 3.5

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Definition of Expected Value

Definition: Suppose a discrete random variable X has probability density function $p_X(k)$.

The **expected value** of X, denoed by $E(X)$ or μ or μ_X is given by:

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The sum is taken over all values k that the random variable can assume.

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Equivalently, integral may also be taken over the subset of the real line where f_Y has support.

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Expected Value of a Function

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Expected Value of a Linear Function

For any random variable W and any constants a and b,

$$E(aW + b) \quad = \quad a \cdot E(W) + b$$

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Example: If X is the outcome of rolling a single die, $m = 3.5$.

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Equivalently, if $F_Y(y)$ is the cdf of Y, the median is the number m that satisfies the equation

$$F_Y(m) = 0.5$$