Larson and Marx Section 3.3

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Discrete Probability Functions

Suppose S is a sample space which is either finite or countably infinite. Then we have the following definition:

Definition: A **discrete probability function** p is a real-valued function defined for any element of S such that:

a. $0 \leq p(s)$ for each $s \in S$

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With this definition, the probabilities of events will satisfy the Kolmogorov axioms.

Discrete Random Variables

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Random variables replace outcomes with real numbers, and generally

result in a much smaller "sample space".

Probability Density Function

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$$p_X(k) = P(X = k)$$

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The value of the pdf is defined to be zero for any value of k that is not in the range of X.

Binomial Probability Density Function

Example If X is a random variable having the binomial distribution, then

$$p_X(k) = P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}, \quad k = 0, 1, \ldots, n$$

Hypergeometric Probability Density Function

Example If X is a random variable having the binomial distribution, then

$$p_X(k) = P(X = k) = \frac{\binom{r}{k}\binom{w}{n-k}}{\binom{r+w}{n}}$$

for values of k for which all quantities are defined.

The Cumulative Distribution Function

Definition:

The **cumulative distribution function** (cdf) of a discrete random variable X, $F_X(t)$ is the probability that X assumes a value less than or equal to t:

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The Cumulative Distribution Function

Example: Suppose the random variable X represents the number of successes in 10 independent trials each with a probability of success equal to 0.4.

The probability of 2 or fewer successes is:

$$F_X(2) = P(X \leq 2) = \sum_{k=0}^{2} \binom{10}{k} (.4)^k (.6)^{10-k}$$

Linear Transformations

Theorem: (3.3.1) Suppose X is a discrete random variable with associated probability density function $p_X(x)$, and for some constants a and b, the random variable Y is defined by

$$Y = aX + b$$

Then the probability density function $p_Y(y)$ associated with Y is

$$p_Y(y) = P(Y = y) = P(y = aX + b) = P\left(X = \frac{y-b}{a}\right) = p_X\left(\frac{y-b}{a}\right)$$