Section 3.11: Conditional Densities

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Conditional pdf of a Discrete Random Variable

Definition: (3.11.1) Let X and Y are discrete random variables. The **conditional probability density function of** Y **given that** $X = x$ is given by:

$$p_{Y|x} = P(Y = y \mid X = x) = \frac{p_{XY}(x, y)}{p_X(x)}$$

for $p_X(x) \neq 0$.

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If X, Y, and Z are discrete random variables with joint pdf $P_{XYZ}(x, y, z)$, the **joint conditional pdf** of X and Y given that $Z = z$ is:

$$p_{XY|z}(x, y) \;=\; \frac{p_{XYZ}(x, y, z)}{p_Z(z)}$$

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If X, Y, and Z are discrete random variables with joint pdf $P_{XYZ}(x, y, z)$, the **joint conditional pdf** of X and Y given that $Z = z$ is:

$$p_{XY|z}(x, y) = \frac{p_{XYZ}(x, y, z)}{p_Z(z)}$$

It is also possible to condition on more than one variable. If X, Y, and Z are discrete random variables with joint pdf $P_{XYZ}(x, y, z)$, and the joint (marginal) pdf of Y and Z is $P_{YZ}(y, z)$, the **conditional pdf** of X given that $Y = y$ and $Z = z$ is:

$$p_{X|yz}(x) = \frac{p_{XYZ}(x, y, z)}{p_{YZ}(y, z)}$$

Conditional pdf of a Continuous Random Variable

Definition: (3.11.1) Let X and Y are continuous random variables. The **conditional probability density function of** Y **given that** $X = x$ is given by:

$$f_{Y|x} = \frac{f_{XY}(x, y)}{f_X(x)}$$

Conditional pdf of a Continuous Random Variable

Definition: (3.11.1) Let X and Y are continuous random variables. The **conditional probability density function of Y given that $X = x$** is given by:

$$f_{Y|x} = \frac{f_{XY}(x, y)}{f_X(x)}$$

This definition follows from the derivation of the "conditional" cdf, defined by

$$P(Y \leq y \mid X = x) = \int_{-\infty}^{y} \frac{f_{XY}(x, u)}{f_X(x)} \, du$$