Combinatorial Probability

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In many problems that arise in probability, $N(A)$ and $N(S)$ are obtained using combinatorics and/or the multiplication rule.

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which is usually written in $\log_{10}$ form as

$$\log_{10}(n!) \approx \log_{10}(\sqrt{2\pi}) + \left(n + \frac{1}{2}\right) \log_{10}(n) - \log_{10}(e)$$

Probability of a Flush in Poker

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Solution: For each of the four suits, there are

$$\binom{13}{5}$$

flushes. Since there are 52 choose 5 hands, the probability of a flush is

$$\frac{4 \cdot \binom{13}{5}}{\binom{52}{5}} = \frac{4 \cdot 1287}{2,598,960} = .00198$$