The Classical Model

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- 1) The number of possible outcomes is finite
- 2) All outcomes are equally likely

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In what follows, we will characterize this model in the context of the Kolmogorov axioms.

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In the terminology of set theory, the cardinality of the set S is n,

$$N(S) = n$$

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In other words, for any subset A of S, the value of the probability function $P(A)$ is just the ratio of the cardinality of A to the cardinality of S.

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Axiom 1:

$$P(A) \geq 0 \quad \text{for any} \quad A \subseteq S$$

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Axiom 1:

$$P(A) \geq 0 \quad \text{for any} \quad A \subseteq S$$

This axiom is satisfied by P, since for any subset A of S,

$$N(A) \geq 0$$

which implies that

$$P(A) = \frac{N(A)}{N(S)} \geq 0$$

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Axiom 2:

$$P(S) = 1$$

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This axiom is satisfied because in this case

$$P(S) = \frac{N(S)}{N(S)} = 1$$

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Recall that for any two finite sets A and B,

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Recall that for any two finite sets A and B,

$$N(A \cup B) = N(A) + N(B) - N(A \cap B)$$

If A and B are mutually exclusive, $(A \cap B) = \emptyset$, so $N(A \cap B) = 0$ and the formula reduces to:

$$N(A \cup B) = N(A) + N(B)$$

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Axiom 3: Now it follows that

$$P(A \cup B) = \frac{N(A \cup B)}{N(S)} = \frac{N(A) + N(B)}{N(S)} = \frac{N(A)}{N(S)} + \frac{N(B)}{N(S)}$$

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so the equation becomes

$$P(A \cup B) = P(A) + P(B)$$

and the third Kolmogorov axiom is satisfied.