MA395 Takehome Quiz 3

Name:

1) A fair coin is tossed until the first tail appears. The payoffs are:

$$\text{payoff } = \begin{cases} \$2^k & 1 \le k \le 9 \\ \$1,000 & k \ge 10 \end{cases}$$

How much should you have to pay to play the game to make it "fair" (i.e., to make your expected winnings equal to the cost to play the game)?

2) Show that if the random variable Y has the exponential distribution

$$f_Y(y) = \lambda e^{-\lambda x} \qquad y > 0$$

then

$$E(Y) = \frac{1}{\lambda} \quad \text{and} \quad Var(Y) = \frac{1}{\lambda^2}$$

3) If $E(W) = \mu$ and $Var(X) = \sigma^2$ show that

$$E\left(\frac{W-\mu}{\sigma}\right) = 0 \quad \text{and} \quad Var\left(\frac{W-\mu}{\sigma}\right) = 1$$

4) Let Y be a uniform random variable defined over the interval $(0, 2)$. Find an expression for the r^{th} moment of Y about the origin.

5) Suppose a random variable Y has pdf

$$f_Y(y) \quad = \quad c \cdot y^{-6}, \quad y > 1$$

a) Find c.

b) What is the highest moment of Y that exists?