MA395 Takehome Quiz 2

Name:

1) Let n be any positive integer. Show that, for a given n, the function

$$f_Y(y) = (n+2)(n+1)y^n(1-y), \quad 0 \leq y \leq 1$$

is a pdf.

2) Suppose Y is an exponential random variable, so that

$$f_Y(y) = \lambda e^{-\lambda y}, \quad y \geq 0$$

Find the cumulative distribution function $F_Y(y)$.

3) Suppose Y is a random variable representing the time to failure for a machine. The **hazard rate** is defined to be the probability that an item fails at time y, *given that it has survived until time* y. In terms of the pdf and cdf of Y, the hazard rate is:

$$h(y) = \frac{f_Y(y)}{1 - F_Y(y)}$$

Find $h(y)$ if Y has an exponential distribution.

4) Find a constant α so that, for some given value $k > 1$,

$$f_Y(y) = \frac{\alpha}{y^2}, \quad y \geq k$$

is a pdf.

5) Suppose

$$F_Y(y) = \frac{1}{12}(y^2 + y^3), \quad 0 \leq y \leq 2$$

find $f_Y(y)$.