MA395 Takehome Quiz 1

## Name:

1) A baseball player is said to be a " 300 hitter" if he has a $30 \%$ chance of hitting safely each time he bats. If a 300 hitter bats 4 times in a certain game, find the probabilities of getting $n$ hits, for $n \in$ $\{0,1,2,3,4\}$.

Solution: Consider each time at bat as an independent Bernoulli trial with probability of success $p=0.3$. Then:

$$
\begin{aligned}
& p(0 \text { hits })=\binom{4}{0}(0.3)^{0}(0.7)^{4}=0.240 \\
& p(1 \text { hit })=\binom{4}{1}(0.3)^{1}(0.7)^{3}=0.412 \\
& p(2 \text { hits })=\binom{4}{2}(0.3)^{2}(0.7)^{2}=0.265 \\
& p(3 \text { hits })=\binom{4}{3}(0.3)^{3}(0.7)^{1}=0.076 \\
& p(4 \text { hits })=\binom{4}{4}(0.3)^{4}(0.7)^{0}=0.008
\end{aligned}
$$

2) Two fair dice are rolled. Suppose a random variable $X$ is defined to be the product of the two faces that come up.
a) Write a table that shows the values $X$ can assume and the outcomes that produce each of those values (i.e., construct a table that defines $X$ as a function from the sample space into the subset of the real numbers that contains all possible outcomes).
b) Assuming that all 36 possible outcomes are equally likely, find the probability distribution $f(X)$.

Solution: Assume each cell has probability 1/36.
The 18 possible outcomes are:

$$
\{1,2,3,4,5,6,8,9,10,12,15,16,18,20,24,25,30,36\}
$$

These are represented by the following table:

|  | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | 1 | 2 | 3 | 4 | 5 | 6 |
| $\mathbf{2}$ | 2 | 4 | 6 | 8 | 10 | 12 |
| $\mathbf{3}$ | 3 | 6 | 9 | 12 | 15 | 18 |
| $\mathbf{4}$ | 4 | 8 | 12 | 16 | 20 | 24 |
| $\mathbf{5}$ | 5 | 10 | 15 | 20 | 25 | 30 |
| $\mathbf{6}$ | 6 | 12 | 18 | 24 | 30 | 36 |

The associated probabilities are simply the number of cells containing the outcome time $1 / 36$ :

| 1 | $1 / 36$ | 2 | $2 / 36$ | 3 | $2 / 36$ | 4 | $3 / 36$ | 5 | $2 / 36$ | 6 | $4 / 36$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8 | $2 / 36$ | 9 | $1 / 36$ | 10 | $2 / 36$ | 12 | $4 / 36$ | 15 | $2 / 36$ | 16 | $1 / 36$ |
| 18 | $2 / 36$ | 20 | $2 / 36$ | 24 | $2 / 36$ | 25 | $1 / 36$ | 30 | $2 / 36$ | 36 | $1 / 36$ |

3) A squirrel born in a certain year has a $15 \%$ chance of surviving to breed the following year. What size litter is necessary to make the probability that one or more offspring survive to breed at least $2 / 3$ ?

Solution: Let $n$ be the litter size. Consider success to be survival to the following year, the $p=0.15$. The probability of at least one member surviving to breed is:

$$
P_{s}(n)=P(k>0)=\sum_{k=1}^{n}\binom{n}{k}(0.15)^{k}(0.85)^{n-k}=1-(0.85)^{n}
$$

So, we want to find the smallest value of $n$ for which $1-(0.15)^{n} \geq 2 / 3$
Values of $P_{s}(n)$ for various values of $n$ are:

$$
\begin{array}{lll}
n=1 & P_{s}(1)=1-(0.85)^{1} & 0.150 \\
n=2 & P_{s}(2)=1-(0.85)^{2} & 0.278 \\
n=3 & P_{s}(3)=1-(0.85)^{3} & 0.386 \\
n=4 & P_{s}(4)=1-(0.85)^{4} & 0.478 \\
n=5 & P_{s}(5)=1-(0.85)^{5} & 0.556 \\
n=6 & P_{s}(6)=1-(0.85)^{6} & 0.623 \\
n=7 & P_{s}(7)=1-(0.85)^{7} & 0.679
\end{array}
$$

From the above table, the smallest value of $n$ for which $P_{x}(n) \geq 2 / 3$ is 7 .
4) The bluejay population in a certain county is estimated to be 4500 . A sample indicates that $10 \%$ are infected with the West Nile virus. If there are $n$ bluejays in a certain field, write a formula for the probability that $k$ of them are infected with the West Nile virus.

Solution: Assume that we can actually see $n$ bluejays at some point in time, so there is no question that there are $n$ distinct birds. Since the population is assumed to be 4500 birds, we are in effect sampling without replacement from a finite population, so the number of infected birds should follow a hypergeometric distribution, with:

$$
\begin{array}{ll}
\text { Population size: } & N=4500 \\
\text { Infected birds: } & I=450=0.10 \times 4500 \\
\text { Uninfected birds: } & U=4050=4500-I \\
\text { Sample size: } & n=10
\end{array}
$$

The probability that a sample of size $n=10$ contains exactly $k$ infected birds is:

$$
\frac{\binom{I}{k}\binom{U}{10-k}}{\binom{N}{10}}
$$

5) A burglar removes 4 gems at random from a display case that contains 10 real and 25 fake diamonds. What is the probability that the last gem removed is the second real diamond in the set of four that were removed?

Solution: For the fourth gem to be the second real diamond, the first three draws must result in two fake and one real diamond. Then, at the time of the fourth draw, the case will contain 23 fake and 9 real diamonds.

The probability that the first three draws result in one real and two fake diamonds is the hypergeometric probability:

$$
P(x=1)=\frac{\binom{10}{1}\binom{25}{2}}{\binom{35}{3}}=\frac{\frac{10 \cdot 25 \cdot 24}{2}}{\frac{35 \cdot 34 \cdot 33}{3 \cdot 2 \cdot 1}}=\frac{10 \cdot 25 \cdot 24}{35 \cdot 34 \cdot 33}=0.458
$$

Now the probability that the fourth draw yields a real gem is $9 / 32$, so the probability of the event described is

$$
0.458 \cdot \frac{9}{32}=0.129
$$

