MA362 MIDTERM STUDY GUIDE

The midterm examination will be held **Thursday, March 25th**. The exam will consist of three sections:

- A matching section with terms definitions
- Two proofs you have seen before. Two of the following four proofs will appear on the exam:
  - Prove that uniformly continuous functions in $\mathbb{R}^n$ preserve Cauchy sequences
  - Prove that if a sequence $\{x_n\}$ in a metric space (X, ρ) is convergent, then it is bounded.
  - Prove that if the function $f : (X, \rho) \setminus \{a\} \to (Y, \tau)$ has the property that for every sequence $\{x_n\} \in X \setminus \{a\}$ that converges to a, the sequence $\{f(x_n)\}$ converges to $L \in Y$ then
  $$\lim_{x \to a} f(x) = L$$
  - Prove the Bolzano-Weierstrass theorem in $\mathbb{R}^n$: Every bounded sequence $\{x\}$ in $\mathbb{R}^n$ has a convergent subsequence.
- Two proofs you may *not* have seen before but typical of proofs found in lectures and on the homework

(continued)

The terms will be taken from the following list:

Term	Text Reference
linear space	notes
norm	notes
function (as subset of $A \times B$)	notes
Euclidean norm $\|x\|$	Definition 8.3
$\updownarrow^1$ norm $\|x\|_1$	Definition 8.3
sup norm $\|x\|_\infty$	Definition 8.3
Euclidean distance $\|x - y\|$	Definition 8.3
linear function $T : \mathbb{R}^n \to \mathbb{R}^m$	Definition 8.12
operator norm $\|T\|$, $T \in \mathcal{L}(\mathbb{R}^n, \mathbb{R}^m)$	Definition 8.16
convergent sequence	Definitions 9.1,10.13
Cauchy sequence	Definition 9.1,10.13
bounded sequence	Definition 9.1,10.13
complete metric space	Definition 10.19
cluster point	Definition 10.22
open ball	Definitions 8.19,10.7
open set	Definitions 8.20,10.8
closed set	Definitions 8.20,10.8
function limit	Definitions 9.14,10.25
continuous function	Definitions 9.23,10.27
uniformly continuous function	Definition 9.24,10.51
Bolzano-Weierstrass Property	Definition 10.30
compact set	Definition 9.10,10.42
metric	Definition 10.1
metric space	Definition 10.1
discrete metric	Example 10.3