MA362 FINAL EXAM STUDY GUIDE

The final examination will be held **Thursday, May 13th**. The exam will consist of three sections:

- A matching section with terms and definitions (including theorems or lemmas that can be stated briefly such as "orthogonality property" matching to $\int_{-\pi}^{\pi} \sin kx \cos jx \, dx = 0$)
- Two proofs you have seen before. Two of the following four proofs will appear on the exam:
  - Prove that if $x_n \to x$ as $n \to \infty$ then $\limsup_{n\to\infty} x_n = x$ (Remark 6.22 iii)
  - Prove that if a trigonometric series converges uniformly to f it is the Fourier series (Theorem 14.4)
  - Prove that if $f : E \subseteq (X, \rho) \to (Y, \tau)$ is continuous and E is compact, then f is uniformly continuous on E (Theorem 10.52)
  - Prove the Uniform Cauchy Criterion (Lemma 7.11)
- Two proofs you may *not* have seen before but typical of proofs found in lectures and on the homework

(continued)

The terms will be taken from the following list:

Term	Text Reference
linear space	notes
norm	notes
function (as subset of $A \times B$)	notes
Euclidean norm $\|x\|$	Definition 8.3
$\updownarrow^1$ norm $\|x\|_1$	Definition 8.3
sup norm $\|x\|_\infty$	Definition 8.3
Euclidean distance $\|x - y\|$	Definition 8.3
linear function $T : \mathbb{R}^n \to \mathbb{R}^m$	Definition 8.12
operator norm $\|T\|$, $T \in \mathcal{L}(\mathbb{R}^n, \mathbb{R}^m)$	Definition 8.16
convergent sequence	Definitions 9.1,10.13
Cauchy sequence	Definition 9.1,10.13
bounded sequence	Definition 9.1,10.13
complete metric space	Definition 10.19
cluster point	Definition 10.22
open ball	Definitions 8.19,10.7
open set	Definitions 8.20,10.8
closed set	Definitions 8.20,10.8
function limit	Definitions 9.14,10.25
continuous function	Definitions 9.23,10.27
uniformly continuous function	Definition 9.24,10.51
Bolzano-Weierstrass Property	Definition 10.30
compact set	Definition 9.10,10.42
metric	Definition 10.1
metric space	Definition 10.1
discrete metric	Example 10.3
power series	Section 7.3
interval of convergence	Definition 7.23
radius of convergence	Definition 7.20
pointwise convergence (function sequence)	Definition 7.1
uniform convergence (function sequence)	Definition 7.1
pointwise convergence (function series)	Definition 7.13
uniform convergence (function series)	Definition 7.13
absolute convergence (function series)	Definition 7.13
absolute convergence (numerical series)	Definition 6.18
conditional convergence (function series)	Definition 6.18

(Continued)

Taylor series remainder term	Definition 7.42
Taylor series	Definition 7.40
Maclaurin series	Definition 7.40
Weierstrass M test	Theorem 7.15
uniform Cauchy criterion	Lemma 7.11
$\limsup x_n$	Definition 6.21
$\liminf x_n$	Definition 6.21
rearrangement	Definition 6.26
root test	Theorem 6.23
ratio test	Theorem 6.24
Cesaro mean	Definition 14.10
Cesaro summable	Definition 14.10
Fourier coefficient	Definition 14.3
Fourier series	Definition 14.3
trigonometric polynomial	Definition 14.1
trigonometric series	Definition 14.1
Euler's identity $e^{i\theta} = \cos\theta + i\sin\theta$	
Algebra in $\mathcal{C}(\mathcal{X})$	Definition 10.65
Uniformly dense in $\mathcal{C}(X)$	Definition 10.66
Uniformly closed in $\mathcal{C}(X)$	Definition 10.66
Algebra that separates points in $\mathcal{C}(X)$	Definition 10.68
$\mathcal{C}^\infty(X)$ the infinitely differentiable functions on X	
Real analytic	Definition 7.38