1. ASSIGNMENT 9

1.1. **Problem 1.**

Suppose $\{x_n\}$ is a real sequence and

$$\liminf_{n\to\infty} x_n = L \in \mathbb{R}$$

If $C \in \mathbb{R}$ is a cluster point of x_n, prove that $C \geq L$.

1.2. **Problem 2.**

Supose $\{x_n\}$ is a real sequence with $x_n \to x$ as $n \to \infty$. Show that

$$\liminf_{n\to\infty} x_n = x$$

1.3. **Problem 3.**

Let Let $\{x_n\}$ be a real sequence. Prove that if

$$\liminf_{n\to\infty} x_n > x \in \mathbb{R} \quad \text{then} \quad x_n > x \quad \text{when } n \text{ is large}$$

1.4. **Problem 4.**

If the statement

$$\text{If} \quad \sum_{k=1}^{\infty} a_k \quad \text{converges absolutely and} \quad a_k \to 0 \text{ as } k \to \infty$$

then

$$\limsup_{k\to\infty} |a_k|^{1/k} < 1$$

is true prove it. If it is false, give a counterexample.