1. ASSIGNMENT 8

1.1. **Problem 1.** Prove Theorem 6.16 i): If $a_k \geq 0$, $b_k > 0$ for large k, and $\lim_{k\to\infty} a_k/b_k \in (0,\infty)$, then

$$\sum_{k=1}^{\infty} a_k \quad \text{converges if and only if} \quad \sum_{k=1}^{\infty} b_k \text{ converges}$$

1.2. **Problem 2.** Suppose a_k and b_k are nonnegative for all $k \in \mathbb{N}$. Prove that

$$\text{If } \sum_{k=1}^{\infty} a_k \quad \text{and} \quad \sum_{k=1}^{\infty} b_k \text{ converge, then } \sum_{k=1}^{\infty} a_k b_k \text{ converges}$$

1.3. **Problem 3.** Let

$$s_n = \sum_{k=1}^{n} \frac{(-1)^{k+1}}{k} \quad n \in \mathbb{N}$$

Prove that s_{2n} is strictly increasing, s_{2n+1} is strictly decreasing, and $s_{2n+1} - s_{2n} \to 0$ as $n \to \infty$.

1.4. **Problem 4.** Suppose $a_k \geq 0$ for k sufficiently large and $\sum a_k/k$ converges. Prove that

$$\lim_{j\to\infty} \sum_{k=1}^{\infty} \frac{a_k}{j+k} = 0$$