1. ASSIGNMENT 7

1.1. **Problem 1.** A point a in a metric space X is said to be *isolated* if and only if there is an $r > 0$ sufficiently small that $B_r(a) = \{a\}$. Show that a point $a \in X$ is *not* a cluster point of X if and only if a is isolated.

1.2. **Problem 2.** Prove that a is a cluster point for some $E \subseteq X$ if and only if there is a sequnce $x_n \in E \setminus \{a\}$ such that $x_n \to a$ as $n \to \infty$.

1.3. **Problem 3.** A metric space X has the Bolzano-Weierstrass Property if and only if every bounded sequence $x_n \in X$ has a convergent subsequence. If X has the Bolzano-Weierstrass property, $E \subseteq X$ is closed and bounded and $f : E \to \mathbb{R}$ is continuous on E, prove that f is bounded on E. (Hint: see the proof of Theorem 3.26)

1.4. **Problem 4.** Suppose a metric space X has the Bolzano-Weierstrass Property, $E \subseteq X$ is closed and bounded and $f : E \to \mathbb{R}$ is continuous on E. Prove that there exist points $x_m, x_M \in E$ such that

$$f(x_M) = \sup_{x \in E} \quad \text{and} \quad f(x_m) = \inf_{x \in E} f(x)$$