1. ASSIGNMENT 6

1.1. **Problem 1.** Let $\{x_n\}$ be Cauchy in X. Prove that $\{x_n\}$ converges if and only if at least one of its subsequences converges.

1.2. **Problem 2.** $E \subseteq X$ is said to be *sequentially compact* if and only if every sequence x_n in E has a convergent subsequence whose limit belongs to E. Prove that every sequentially compact set is closed and bounded.

1.3. **Problem 3.** Prove that $\{x_k\}$ is bounded in X if and only if

$$\sup_{k \in \mathbb{N}} \rho(x_k, a) < \infty \quad \forall a \in X$$

1.4. **Problem 4.** Prove that the set of continuous real-valued functions on an interval $[a, b]$, $\mathcal{C}[a, b]$ together with the distance function induced by the norm

$$\|f\|_1 = \int_a^b |f(x)|\, dx$$

is a metric space, but not a complete metric space (hint: see Remark 7.3 for a counterexample)