1. ASSIGNMENT 5

1.1. Problem 1.

Problem 1. Determine whether the following limit exists. If it does not exist, explain why not, and if it does exist, find the limit.

$$\lim_{x\to(0,0)} \frac{\sin x \sin y}{x^2+y^2}$$

(Hint: consider what happens as x approaches $(0,0)$ along the line $x=y$ and along the x and y axes).

1.2. Problem 2.

Problem 2. (Sequential characterization of limits) Prove Theorem 9.5 part ii):

Theorem. *Let $a \in \mathbb{R}^n$, $V \subseteq \mathbb{R}^n$ be an open set containing a, and $f : V \setminus a \to \mathbb{R}^m$ a function. Then*

$$L = \lim_{x\to a}$$

exists if and only if $f(x_k) \to L$ as $k \to \infty$ for every sequence $\{x_k\} \in V \setminus \{a\}$ that converges to a as $k \to \infty$.

1.3. Problem 3.

Problem 3. Prove that uniformly continuous functions in $\mathbb{R}^n$ preserve Cauchy sequences. (hint: See Lemma 3.38)

1.4. Problem 4.

Problem 4. (9.48)

For $D \subseteq E \subseteq \mathbb{R}^n$ suppose D is dense in E, that is, $\overline{D} = E$. If $f : D \to \mathbb{R}^m$ is uniformly continuous on D, prove that f has a continuous extension

$$g : E \to \mathbb{R}^m \quad \text{such that} \quad g(x) = f(x) \quad \forall x \in D$$

(hint: use the result of problem 3 and see Theorem 3.40)