1. ASSIGNMENT 3

1.1. Problem 1.

Using the definition of a convergent sequence in $\mathbb{R}^n$, show that the following sequence converges:

$$x_k = \left(\frac{1}{k}, 1 - \frac{1}{k^2}\right)$$

1.2. Problem 2.

Suppose x_n and y_n are sequences in $\mathbb{R}^n$ with $x_n \to \vec{0}$ and y_n bounded. Show that the sequence of inner products

$$x_n \cdot y_n$$

converges to zero.

1.3. Problem 3.

A subset E of $\mathbb{R}^n$ is called **sequentially compact** if and only if every sequence $x_k \in E$ has a convergent subsequence whose limit is in E.

Prove that $\mathbb{R}^n$ is *not* sequentially compact.

1.4. Problem 4.

Let $E \subseteq \mathbb{R}^n$. A point $a \in \mathbb{R}^n$ is called a **cluster point** of E if, for every $r > 0$,

$$B_r(a) \quad \text{contains infinitely many points of } E \quad \text{for any } r > 0$$

Prove that every bounded infinite subset of $\mathbb{R}^n$ has at least one cluster point. (Hint: a bounded infinite subset must have an infinite number of *distinct* points).