1. ASSIGNMENT 10

1.1. **Problem 1.** Show that if $E = (0, 1)$, the sequence of functions $f_n : E \to \mathbb{R}$ defined by

$$f_n(x) = \frac{1}{nx}, \quad n \in \mathbb{N}$$

converges to $f(x) = 0$ pointwise, but not uniformly on E

1.2. **Problem 2.** Supose $f : \mathbb{R} \to \mathbb{R}$ is uniformly continuous and $\{y_n\}$ is a real sequence that converges to zero as $n \to \infty$. Show that the sequence of functions f_n defined by:

$$f_n(x) = f(x + y_n) \quad x \in \mathbb{R},\ n \in \mathbb{N}$$

converges uniformly to f on $\mathbb{R}$

1.3. **Problem 3.** Let $E \subseteq \mathbb{R}$ be a nonempty set and $f_n : E \to \mathbb{R}$ a sequence of functions that converge to f uniformly on E:

$$f_n(x) \to f(x) \quad \text{uniformly as} \quad n \to \infty$$

Show that uniform convergence preserves uniform continuity, that is, if each f_n is uniformly continuous on E, then so is f.

1.4. **Problem 4.** Suppose $f_n \to f$ and $g_n \to g$ as $n \to \infty$ uniformly on $E \subseteq \mathbb{R}$. Prove that if $f : E \to \mathbb{R}$ and $g : E \to \mathbb{R}$ are bounded on $E \subseteq \mathbb{R}$, then

$$f_n g_n \to fg \quad \text{uniformly on } E$$

Hint: Use the results of problem 7.1.3. You do not have to prove them.