Sets

Gene Quinn

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Practically speaking, a set is such a basic concept that attempts to define it amount to simply giving a synomym.

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Given a set and an arbitrary object, we must be able to decide whether the object belongs to the set or not.

In this case the set is said to be **well defined**

Notation

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A **lower case** letter will designate a member or *element* of a set.

The symbol $\in$ is used to indicate that an element belongs to a set:

$x \in S$ is read:

- "x belongs to S"
- or
- "x is in S"
- or
- "x is an element of S"

Describing Sets

One way of describing a set is to list its elements surrounded by *curly brackets* $\{\}$: The set of natural numbers less than 10 would be described by the notation

$$S = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

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If ellipses are used, it should be obvious what the pattern is:

$$O = \{1, 3, 5, 7, \ldots\} \quad E = \{2, 4, 6, 8, \ldots\}$$

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This is known as set builder notation.

Equality of Sets

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Formally,

$$S = T \quad \text{if and only if} \quad (x \in S \text{ if and only if } x \in T)$$

The Universal Set

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$$S = \{n \mid n \in \mathbb{N} \quad \text{and} \quad n < 10\}$$

In other cases, it will be understood to be, say, the real numbers, from the context:

$$T = \{x \mid x < 5\}$$

usually means

$$T = \{x \mid x \in \mathbb{R} \quad \text{and} \quad x < 5\}$$

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The null set is denoted by the symbol $\emptyset$.

Set Intersection

The **intersection** of two sets, denoted by $\cap$, is the set that consists of all elements belonging to both sets:

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In set builder notation,

$$S \cap T = \{x \mid x \in S \quad \text{and} \quad x \in T\}$$

Set Union

The **union** of two sets, denoted by $\cup$, is the set that consists of all elements that belong to at least one of the sets:

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Subsets

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The **complement** of a set S, denoted by S^c, is the set of all elements in the universal set $\mathcal{U}$ that **do not** belong to S.

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In set builder notation,

$$S^c = \{x \mid x \in \mathcal{U} \quad \text{and} \quad x \notin S\}$$

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In set builder notation,

$$S \setminus T \; = \; \{x \mid x \in S \quad \text{and} \quad x \notin T\}$$