Preview of Proof

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The sequence of statements uses logic to combine the prior results, and the end result is a proof.

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The letters a and b are placeholders in the expression

$$(x - a)(x - b) \ = \ 0$$

Logical Connectives

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The five basic logical connectives and their symbols are:

not	$\sim$
and	$\wedge$
or	$\vee$
if-then	$\Rightarrow$
if and only if	$\Leftrightarrow$

Generalizations

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A generalization implicitly contains a statement like "for all" or "for every".

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Counterexamples

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The generalization

$$x^2 > 4$$

is false by counterexample since $1^2 = 1 < 4$.

Existence Statements

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There exists a real number x such that $x^2 = 0$

Quantifiers

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Two quantifiers are **for all** and **there exists**

Translation

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Translation refers to the process of replacing those terms with their definitions, and is usually one of the first steps in a proof.

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Converting to a logical form makes it easier to reorganize a proof.