1. ASSIGNMENT 4

1.1. **Problem 1.** (1.2.5 a) Prove that

$$\text{if} \quad a > 2 \quad \text{and} \quad b = 1 + \sqrt{a-1}, \quad \text{then} \quad 2 < b < a$$

Hint: Equation (6) on page 9 may be useful.

1.2. **Problem 2.** (1.3.6 a,b) Prove the approximation and completeness properties for infima:

a) If $E \subset \mathbb{R}$ has a finite infimum and $\epsilon > 0$ is any positive number, then there is a point $a \in E$ such that

$$\inf E + \epsilon > a \geq \inf E$$

b) If $E \subseteq \mathbb{R}$ in nonempty and bounded below, then E has a (finite) infimum.

1.3. **Problem 3.** (1.4.1 a) Suppose $\{x_n\}_{n=1}^{\infty}$ is a recursively generated sequence that for a given initial value x_1 is defined by the formula

$$x_{n+1} = 1 + \sqrt{x_n - 1}, \quad n = 1, 2, 3, \ldots$$

(i.e., we have to be given x_1, then we use the recursion formula to calculate x_2, then x_3, and so on).

Show that if $x_1 > 2$, then

$$2 < x_{n+1} < x_n \quad \forall n \in \mathbb{N}$$

1.4. **Problem 4.** (based on 1.6.0 b) A *dyadic rational* is a real number $x \in \mathbb{R}$ such that

$$x = \frac{n}{2^m} \quad \text{for some} \quad n \in \mathbb{Z} \quad \text{and} \quad m \in \mathbb{N}$$

Prove that the set of all dyadic rationals is **not** uncountable.