

ASSIGNMENT 9

Problem 1. Prove that $E \subseteq \mathbb{R}$ is open if and only if its complement $E^c = \mathbb{R} \setminus E$ is closed.

Problem 2. Prove that every finite set $E \subset \mathbb{R}$ consists entirely of isolated points.

Problem 3. Suppose $E \subseteq \mathbb{R}$ has the following property: For any two nonempty disjoint sets A and B such that $E = A \cup B$, there always exists a sequence $x_n \rightarrow x$ with all x_n contained in one of the sets, and x in the other. Show that E is connected. (Hint: Consider a contrapositive argument - assume E is disconnected and try to show that there exists a pair of nonempty disjoint sets A and B with the property that $E = A \cup B$ and no sequence in A has its limit in B and vice-versa).

Problem 4. Prove the second part of the Heine-Borel Theorem: If $E \subseteq \mathbb{R}$ is closed and bounded, then E is compact.