ASSIGNMENT 8

Problem 1. (3.4.4a) Suppose $f : [0, \infty) \to \mathbb{R}$ is continuous and there is an $L \in \mathbb{R}$ such that

$$f(x) \to L \quad \text{as} \quad x \to \infty$$

Prove that f is uniformly continuous on $[0, \infty)$.

Problem 2. (3.4.6a) Let I be a bounded interval (open, closed, or half-open). Prove that if $f : I \to \mathbb{R}$ is uniformly continuous on I, then f is bounded on I.

Problem 3. (3.4.6b) Show by counterexample that the preceding statement is false if either I is unbounded or f is continuous, but not uniformly continuous on I.

Problem 4. (3.3.4) A point $x \in \mathbb{R}$ is called a *fixed point* of $f : \mathbb{R} \to \mathbb{R}$ if

$$f(x) = x$$

Suppose $f : [a, b] \to [a, b]$ is continuous. Prove that f has a fixed point, that is, there exists $c \in [a, b]$ such that $f(c) = c$.