1. ASSIGNMENT 7

1.1. **Problem 1.** (2.4.1) Prove that if $\{x_n\}$ is a sequence that satisfies

$$|x_n| \leq \frac{2n^2+3}{n^3+5n^2+3n+1}$$

for all $n \in \mathbb{N}$, then $\{x_n\}$ is Cauchy.

1.2. **Problem 2.** (2.4.3b) Suppose $\{x_n\}$ and $\{y_n\}$ are Cauchy sequences in $\mathbb{R}$. Without using Theorem 2.29, prove that $\{x_n + y_n\}$ is Cauchy.

1.3. **Problem 3.** (3.1.1 d) Use Definition 3.1 to prove that

$$\lim_{x \to 0} x^3 \sin(e^{x^2}) \;=\; 0$$

1.4. **Problem 4.** (3.2.4 a) Prove the following comparison theorem for real functions f and g, and $a \in \mathbb{R}$

If $\quad f(x) \geq g(x) \quad$ and $\quad g(x) \to \infty$ as $x \to a \quad$ then $\quad f(x) \to \infty$ as $x \to a$

1.5. **Problem 5.** (3.2.4 b) Prove the following comparison theorem for real functions f, g, and h:

If $f(x) \leq g(x) \leq h(x)$ and

$$\lim_{x \to \infty} f(x) \;=\; L \;=\; \lim_{x \to \infty} h(x)$$

then $g(x) \to L$ as $n \to \infty$.