1. ASSIGNMENT 5

1.1. Problem 1.

1.1. Problem 1. (2.1.5 a) Let C be a fixed positive constant and $\{b_n\}$ a sequence of nonnegative numbers that converges to zero. If $\{x_n\}$ is a sequence of real numbers satisfying

$$|x_n - a| < Cb_n \quad \text{for sufficiently large } n$$

prove that $x_n \to a$ as $n \to \infty$.

1.2. Problem 2. (2.1.7 a,b,c)

a) Suppose that x_n and y_n converge to the same real number. Prove that $x_n - y_n \to 0$ as $n \to \infty$

b) Prove that the sequence $\{x_n\}$ with $x_n = n$ does not converge.

c) Show by counterexample that the converse of part a) is false, that is, it is possible to have sequences $\{x_n\}$ and $\{y_n\}$ with $x_n \neq y_n$ for all n, such that neither is convergent but $x_n - y_n \to 0$ as $n \to \infty$. (Hint: consider unbounded sequences)

1.3. Problem 3. (2.2.5) Suppose $x \in \mathbb{R}$, $x_n \geq 0$, and $x_n \to x$ as $n \to \infty$. Prove that

$$\sqrt{x_n} \to \sqrt{x} \quad \text{as} \quad n \to \infty$$

For the case $x = 0$, use inequality (8) of Section 1.2.

1.4. Problem 4. (2.2.6) Prove that for any given $x \in \mathbb{R}$ there is a sequence $\{r_n\}$ in $\mathbb{Q}$ that converges to x.