

ASSIGNMENT 10

Problem 1. Show that $E \subseteq \mathbb{R}$ is a G_δ set if and only if E^c is an F_σ set.

Problem 2. Show that the irrationals are dense in the reals (Hint: show that there is an irrational number between any two real numbers. Hint for hint: Start by showing that the sum of a rational number and an irrational is irrational)

Problem 3. A subset $E = \{x\} \subset \mathbb{R}$ consisting of a single point is called a **singleton**. Prove that a singleton is a closed set.

Problem 4. Prove that the set of rationals \mathbb{Q} is an F_σ set and \mathbb{I} , the set of irrationals, is a G_δ set. (Hint: consider the fact that since there is an irrational between every two reals (hence any two rationals), \mathbb{Q} must be a collection of singletons).

Problem 5. Show that it is impossible to write \mathbb{R} as a countable union of closed sets, each of which contains no nonempty open intervals.