MA361 Assignment 1

Name:

Write a proof of the following theorem:

Theorem 1.

 $|x - L| < \epsilon$ if and only if $L - \epsilon < x < L + \epsilon$

For *prior results*, you may assume any of the following:

- The definition of the absolute value function $(|z| = z \text{ if } z \ge 0,$ otherwise |z| = -z)
- The properties of the real numbers as a field (i.e., basic arithmetic)
- The properties of inequalities including transitivity and sense reversal when both sides are multiplied by a negative value

Because the theorem has the biconditional or "if and only if" form, your proof will have two parts:

- First assume $|x L| < \epsilon$ and construct a series of steps ending with $L \epsilon < x < L + \epsilon$, giving a justification for each step.
- Next, assume $L \epsilon < x < L + \epsilon$ and construct a series of steps ending with $|x L| < \epsilon$, again justifying each step.

The absolute value function is defined differently for negative and nonnegative arguments, so you may have to consider each of these cases separately.

You should write the proof in two column format, with the steps in the left hand column, and the justification in the right hand column.