Choose any 8 of the following:

1) Suppose a sequence (x_n) is defined by the recursion formula

$$x_{n+1} = \frac{1}{5 - 2x_n}, \qquad n = 1, 2, 3, \ldots$$

with $x_1 = 1$. Prove that (x_n) converges and

$$\lim x_n = \frac{5 - \sqrt{17}}{4}$$

2) Use the Monotone Convergence Theorem to give a proof of the Nested Interval Property.

3) Use the Bolzano-Weierstrass Theorem to give a proof of the Nested Interval Property.

4) Use the Cauchy Criterion to prove the Bolzano-Weierstrass Theorem.

5) Suppose $A \subseteq \mathbb{R}$ is bounded above. Show that $\sup A \in \overline{A}$

6) If $x = \lim a_n$ for some sequence (a_n) contained in A satisfiying $a_n \neq x$, then x is a limit point of A.

7) If O is an open set and (x_n) is a sequence converging to $x \in O$, prove that all but a finite number of the terms of (x_n) must be contained in O.

8) Show that if $K \subseteq \mathbb{R}$ is compact, then $\sup K$ and $\inf K$ both exist and are elements of K.

9) Show that if K is compact and F is closed, then $F \cap F$ is compact.

10) Show that if $K \subseteq \mathbb{R}$ is closed and bounded, then K is compact.

11) A set E is *totally disconnected* if, given any two points $x, y \in E$, there exist separated sets A and B with

$$x \in A, \quad y \in B, \quad \text{and} \quad E = A \cup B$$

12) Prove that if $A \subseteq \mathbb{R}$ is connected, then $\overline{A}$ is connected.