Span and Image

Gene Quinn

Span

Consider two vectors in $\mathbb{R}^3$:

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad \vec{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

Span

Consider two vectors in $\mathbb{R}^3$:

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \vec{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

If we think of the coordinate axes in $\mathbb{R}^3$ as x, y, and z, we can say that $\vec{v}_1$ lies along the x-axis, and $\vec{v}_2$ lies along the y-axis.

Span

Now think of all of the vectors that lie in the xy plane. The characteristic they have in common is that their z-coordinate is zero.

Span

Now think of all of the vectors that lie in the xy plane. The characteristic they have in common is that their z-coordinate is zero.

In other words, every vector in the xy plane has the form

$$\vec{w} = \begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$$

for some $x, y \in \mathbb{R}$.