Shears

Gene Quinn

Shears

A **shear** is a type of linear transformation

$$T : \mathbb{R}^n \to \mathbb{R}^n$$

that alters one component of the vector it is applied to while leaving the other components unchanged.

The shear transformation adds a multiple k of another component to the component being altered.

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Like all linear transformations, a shear is equivalent to multiplication by some matrix A:

$$T(\vec{v}) \ = \ A\vec{v}$$

for some matrix A.

Vertical Shears

In the special case of two dimensions,

$$T : \mathbb{R}^2 \to \mathbb{R}^2$$

the matrix A of a vertical shear has the form

$$\begin{bmatrix} 1 & 0 \\ k & 1 \end{bmatrix}$$

where k is a **arbitrary** constant (positive or negative).

Vertical Shears

Let's consider the result of the vertical shear transformation on an arbitrary element $x \in \mathbb{R}^2$, (x_1, x_2):

$$T(\vec{x}) = A\vec{x} = \begin{bmatrix} 1 & 0 \\ k & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ kx_1 + x_2 \end{bmatrix}$$

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Evidently the effect of this transformation is to add a multiple of the first component x_1 of $\vec{x}$ to the second component, x_2.

The first component x_1 is left unchanged.

The second component x_2 is replaced by $kx_1 + x_2$.

Vertical Shears

Suppose a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ is defined by:

$$T(\vec{x}) \ = \ A\vec{x}, \quad \text{for all } x \in \mathbb{R}^2$$

where

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

T qualifies as a vertical shear because its associated matrix A has the required form for a vertical shear:

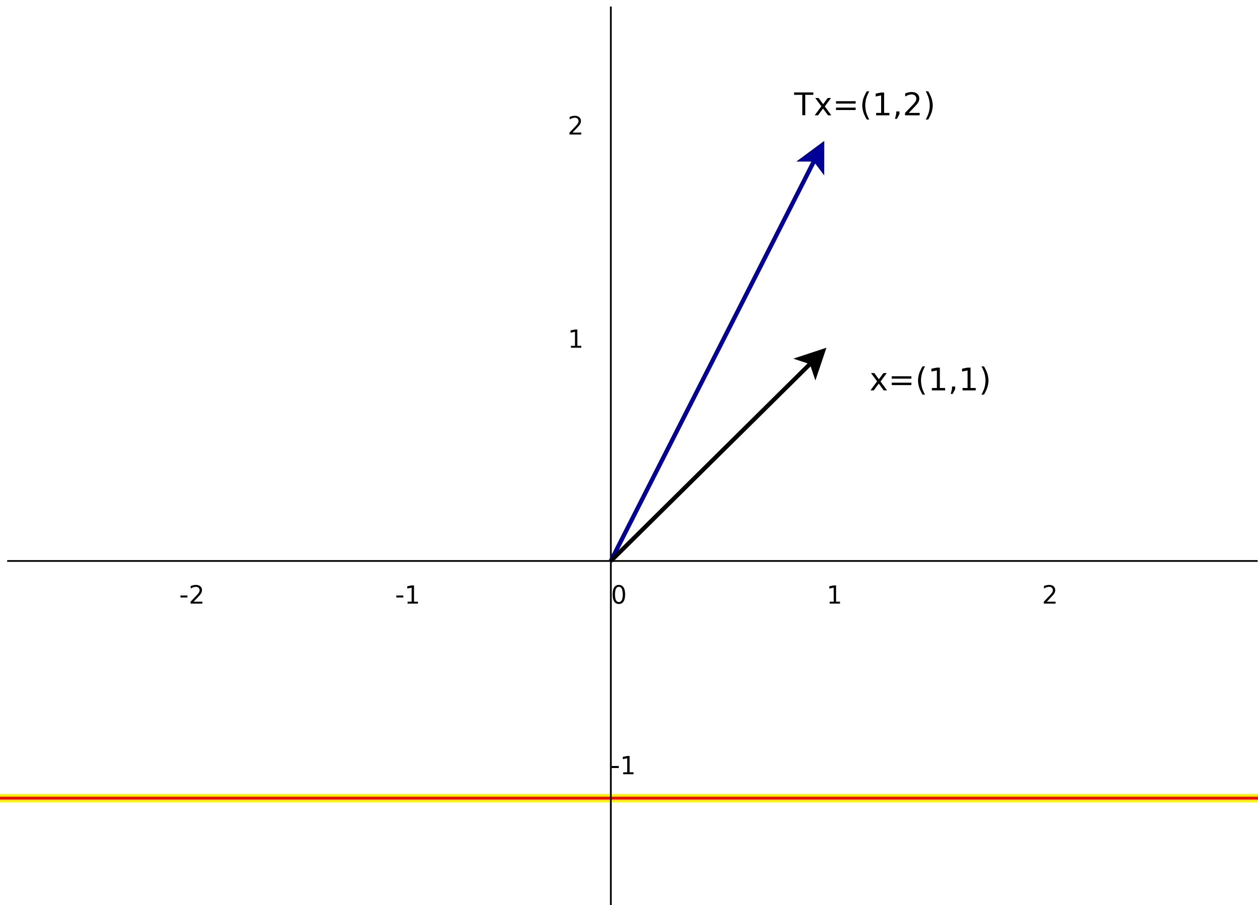
$$\begin{bmatrix} 1 & 0 \\ k & 1 \end{bmatrix}$$

In this case, $k = 1$.

Vertical Shears

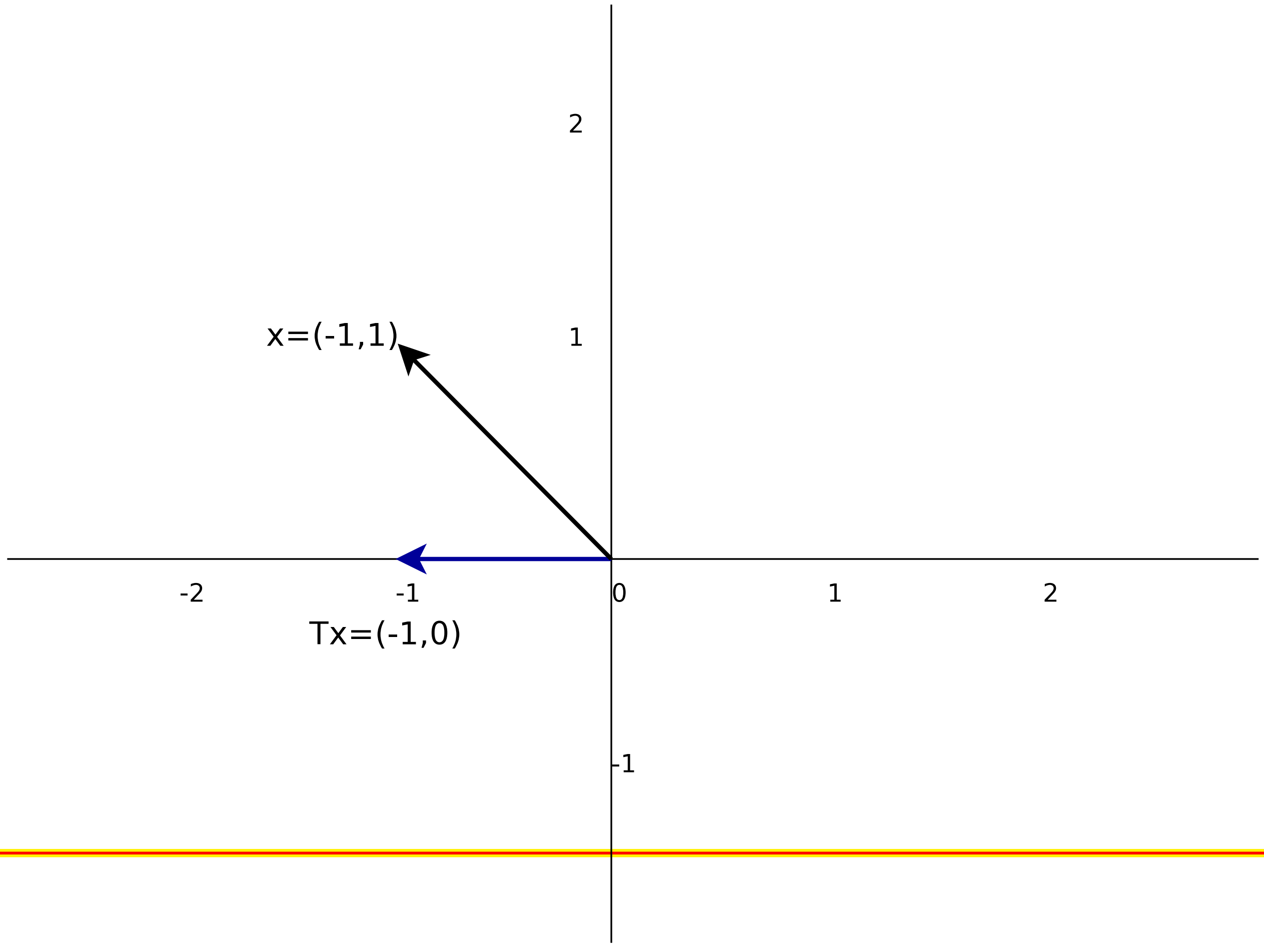
Let's examine the action of T on some vectors in $\mathbb{R}^2$:

$$\text{let } \vec{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \text{ then } T(\vec{x}) = A\vec{x} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$



Vertical Shears

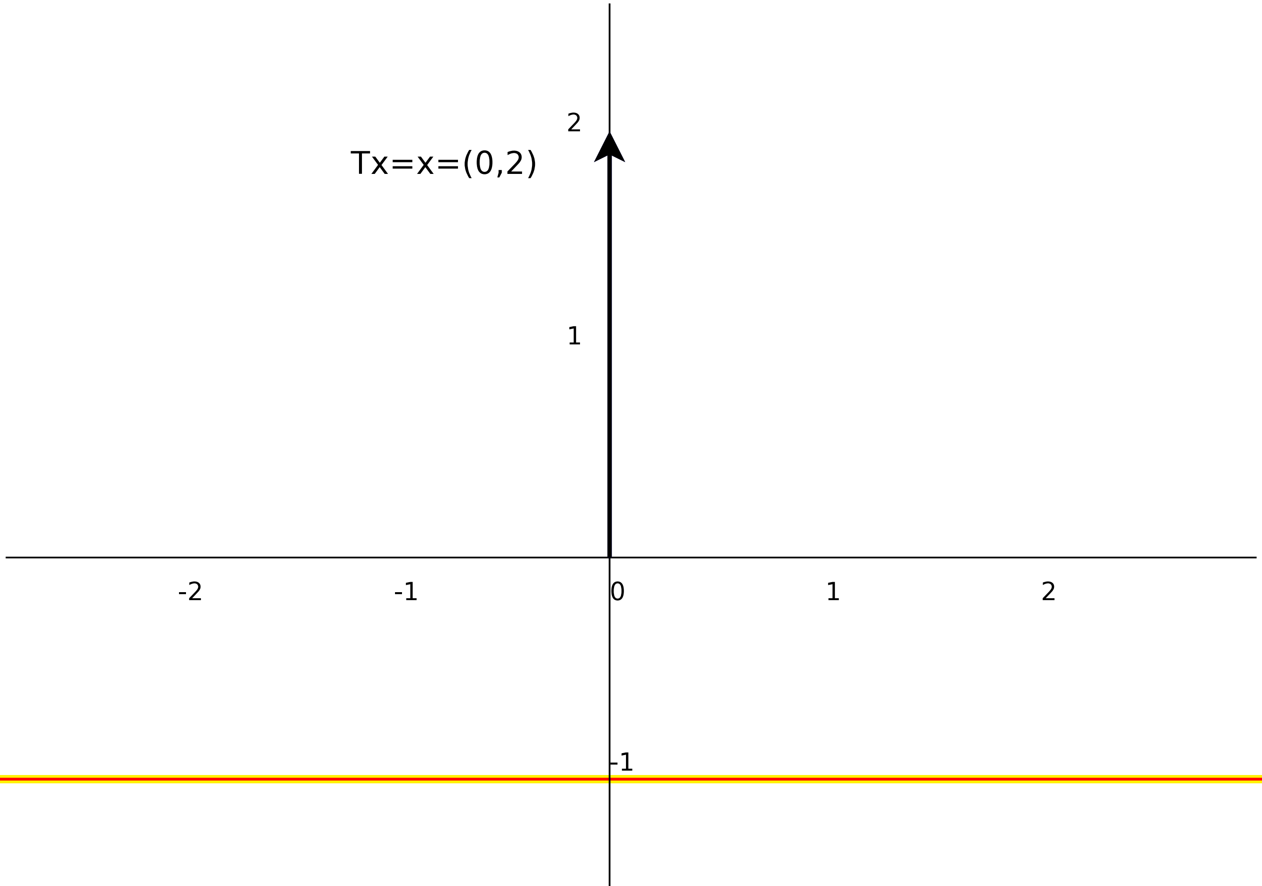
$$\text{let } \vec{x} = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \text{ then } T(\vec{x}) = A\vec{x} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$



Vertical Shears

let $\vec{x} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$ then $T(\vec{x}) = A\vec{x} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$

In this case, $T\vec{x} = \vec{x}$



Vertical Shears

This time suppose a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ is defined by:

$$T(\vec{x}) \;=\; A\vec{x}, \quad \text{for all } x \in \mathbb{R}^2$$

where

$$A = \begin{bmatrix} 1 & 0 \\ -0.5 & 1 \end{bmatrix}$$

T qualifies as a vertical shear because its associated matrix A has the required form for a vertical shear:

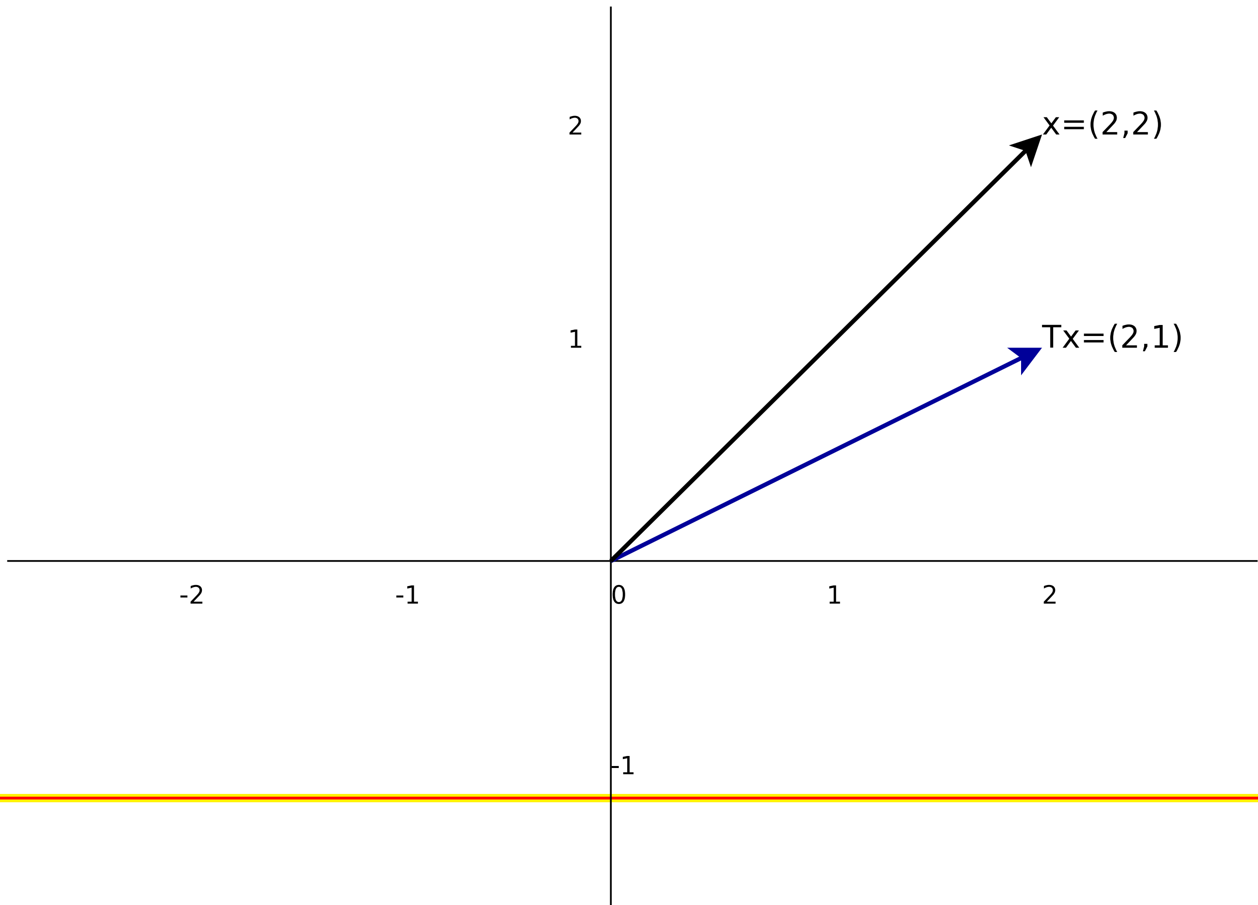
$$\begin{bmatrix} 1 & 0 \\ k & 1 \end{bmatrix}$$

In this case, $k = -0.5$.

Vertical Shears

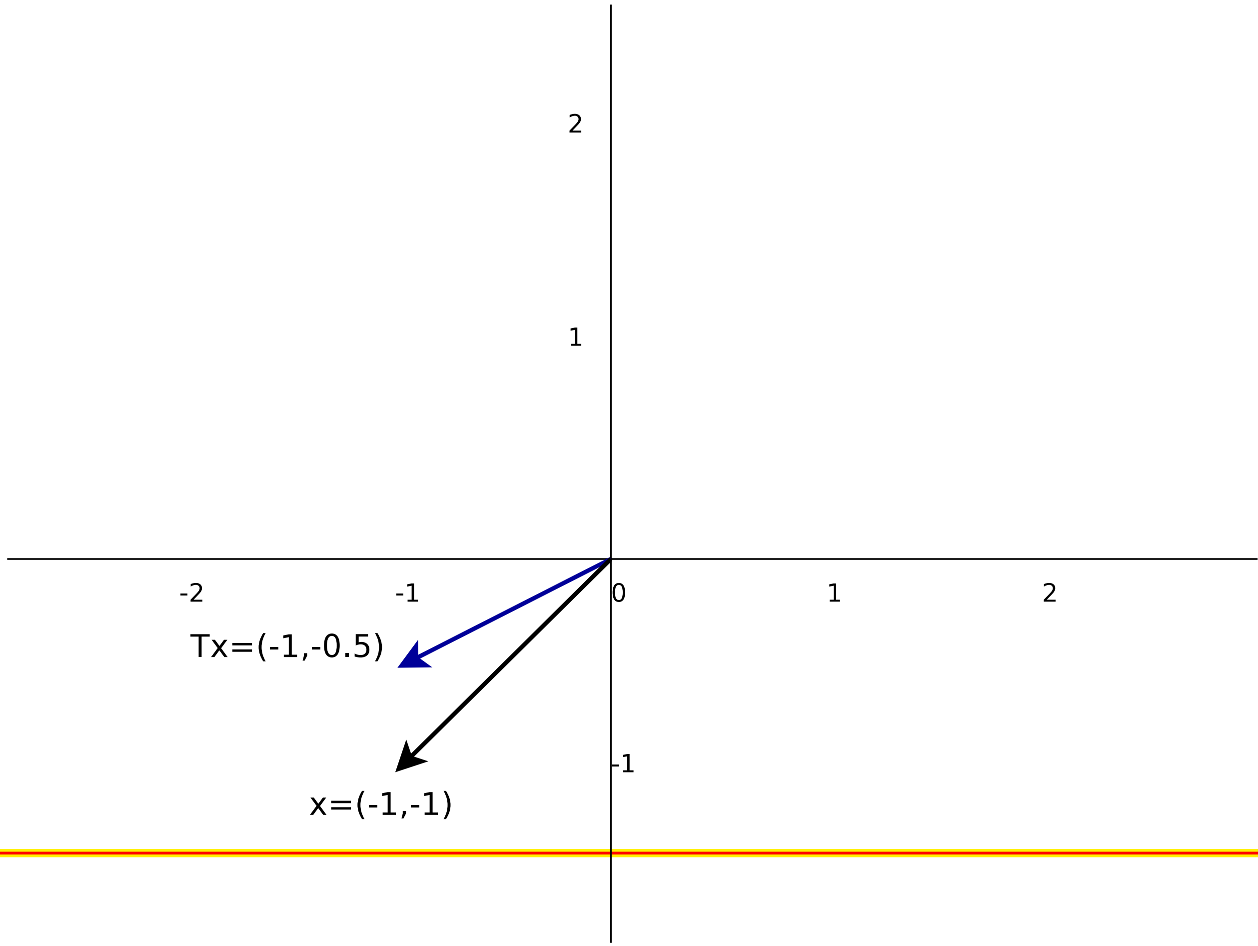
Let's examine the action of T on some vectors in $\mathbb{R}^2$:

$$\text{let } \vec{x} = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \text{ then } T(\vec{x}) = A\vec{x} = \begin{bmatrix} 1 & 0 \\ -0.5 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$



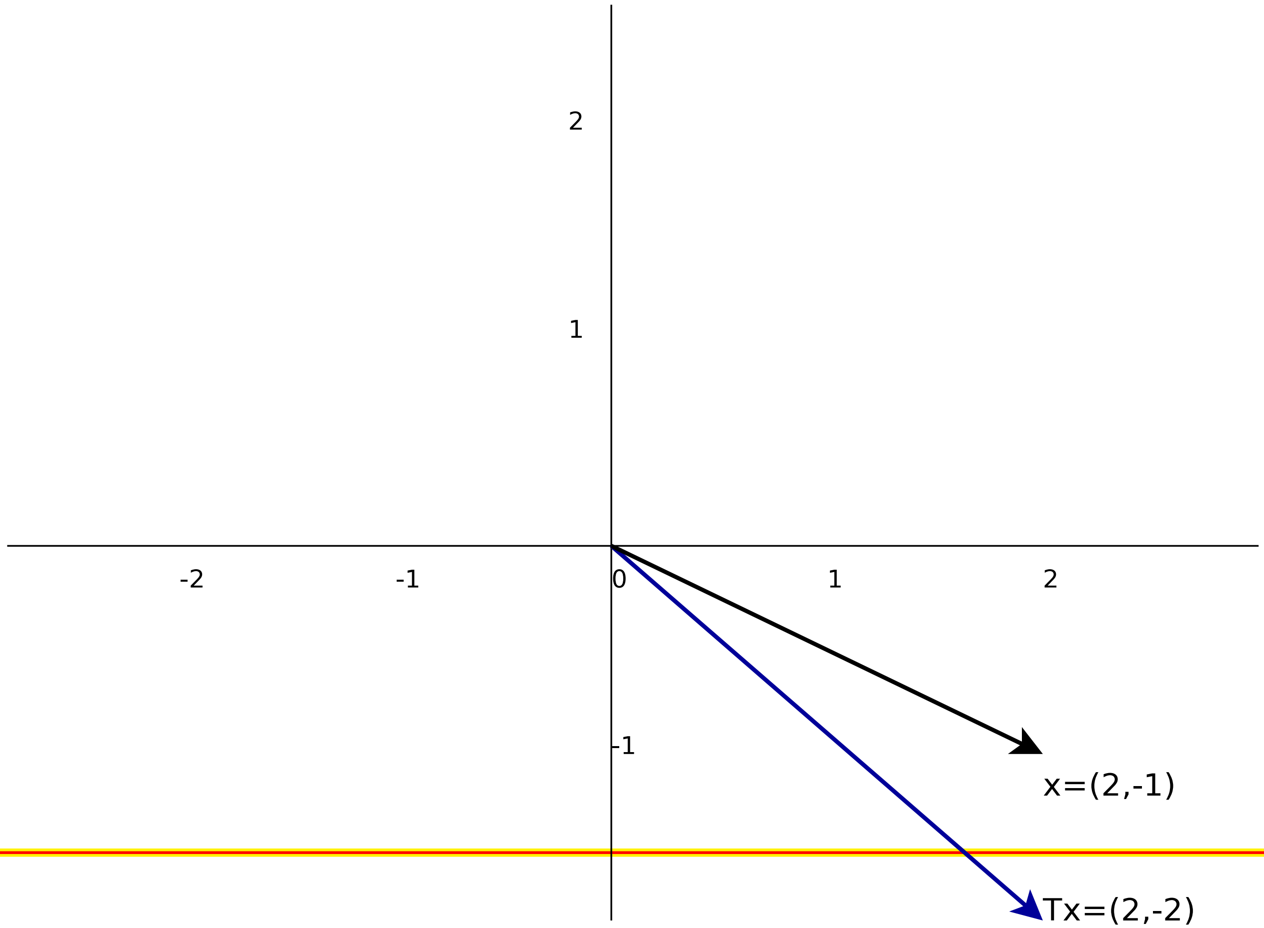
Vertical Shears

let $\vec{x} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$ then $T(\vec{x}) = A\vec{x} = \begin{bmatrix} 1 & 0 \\ -0.5 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ -0.5 \end{bmatrix} = \begin{bmatrix} -0. \\ - \end{bmatrix}$



Vertical Shears

let $\vec{x} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ then $T(\vec{x}) = A\vec{x} = \begin{bmatrix} 1 & 0 \\ -0.5 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$



Horizontal Shears

In the special case of two dimensions,

$$T : \mathbb{R}^2 \to \mathbb{R}^2$$

the matrix A of a horizontal shear has the form

$$\begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$$

where k is a **arbitrary** constant (positive or negative).

Horizontal Shears

Let's consider the result of the horizontal shear transformation on an arbitrary element $x \in \mathbb{R}^2$, (x_1, x_2):

$$T(\vec{x}) = A\vec{x} = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 + kx_2 \\ x_2 \end{bmatrix}$$

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Evidently the effect of this transformation is to add a multiple of the second component x_2 of $\vec{x}$ to the first component, x_1.

The original first component x_1 is replaced by $x_1 + kx_2$.

The second component x_2 is left unchanged.

Horizontal Shears

Suppose a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ is defined by:

$$T(\vec{x}) \;=\; A\vec{x}, \quad \text{for all } x \in \mathbb{R}^2$$

where

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

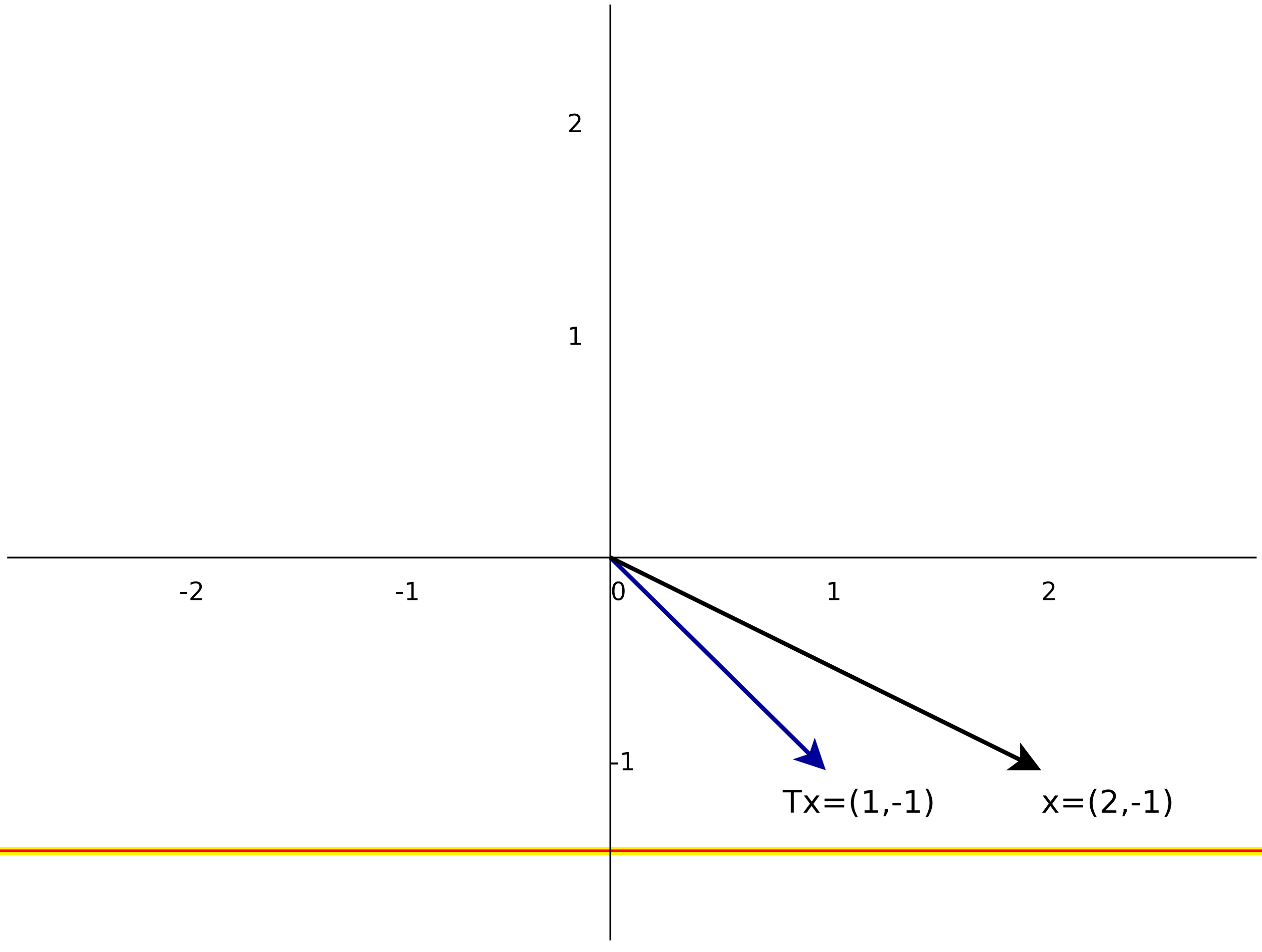
T qualifies as a horizontal shear because its associated matrix A has the required form for a horizontal shear:

$$\begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$$

In this case, $k = 1$.

Horizontal Shears

let $\vec{x} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ then $T(\vec{x}) = A\vec{x} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$



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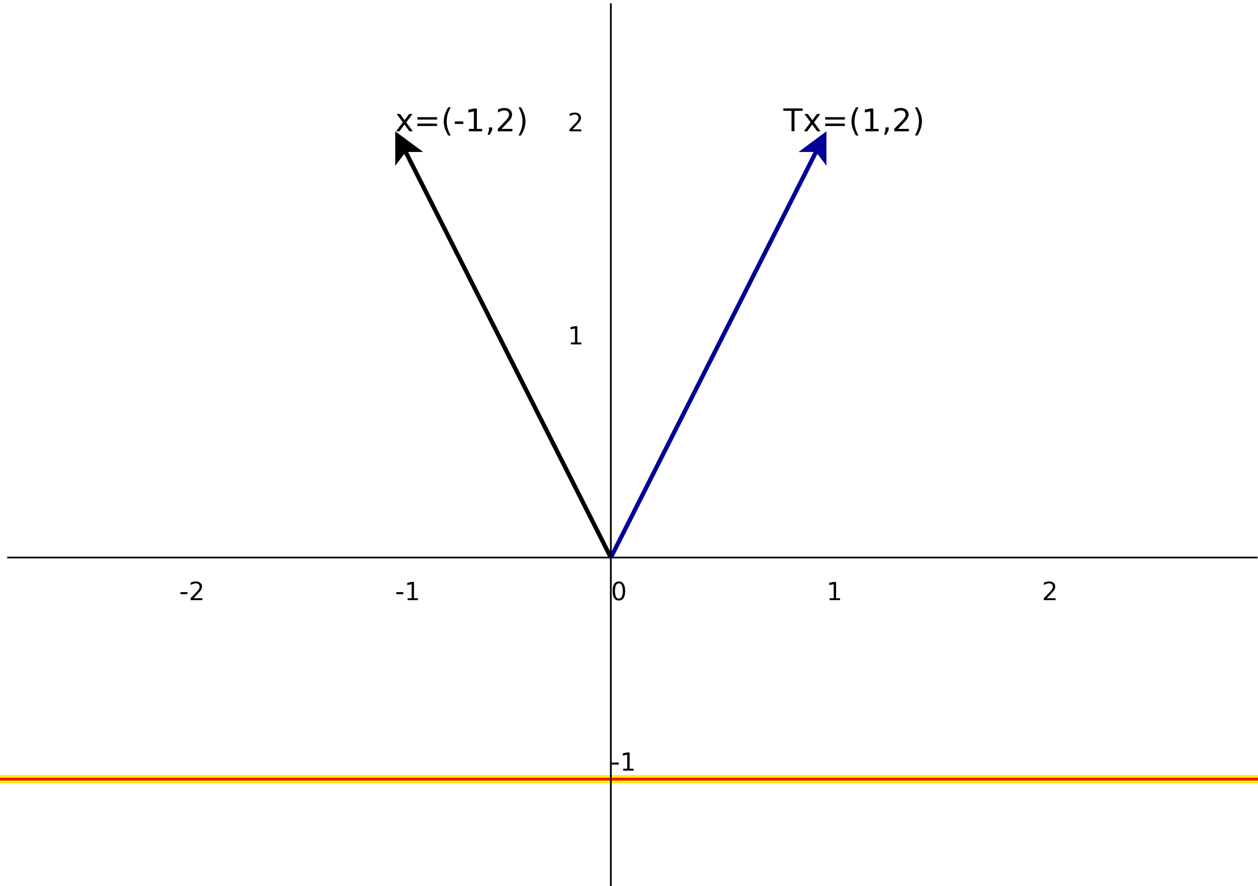
let $\vec{x} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ then $T(\vec{x}) = A\vec{x} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

In this case, $T\vec{x} = \vec{x}$



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where

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T qualifies as a horizontal shear because its associated matrix A has the required form for a horizontal shear:

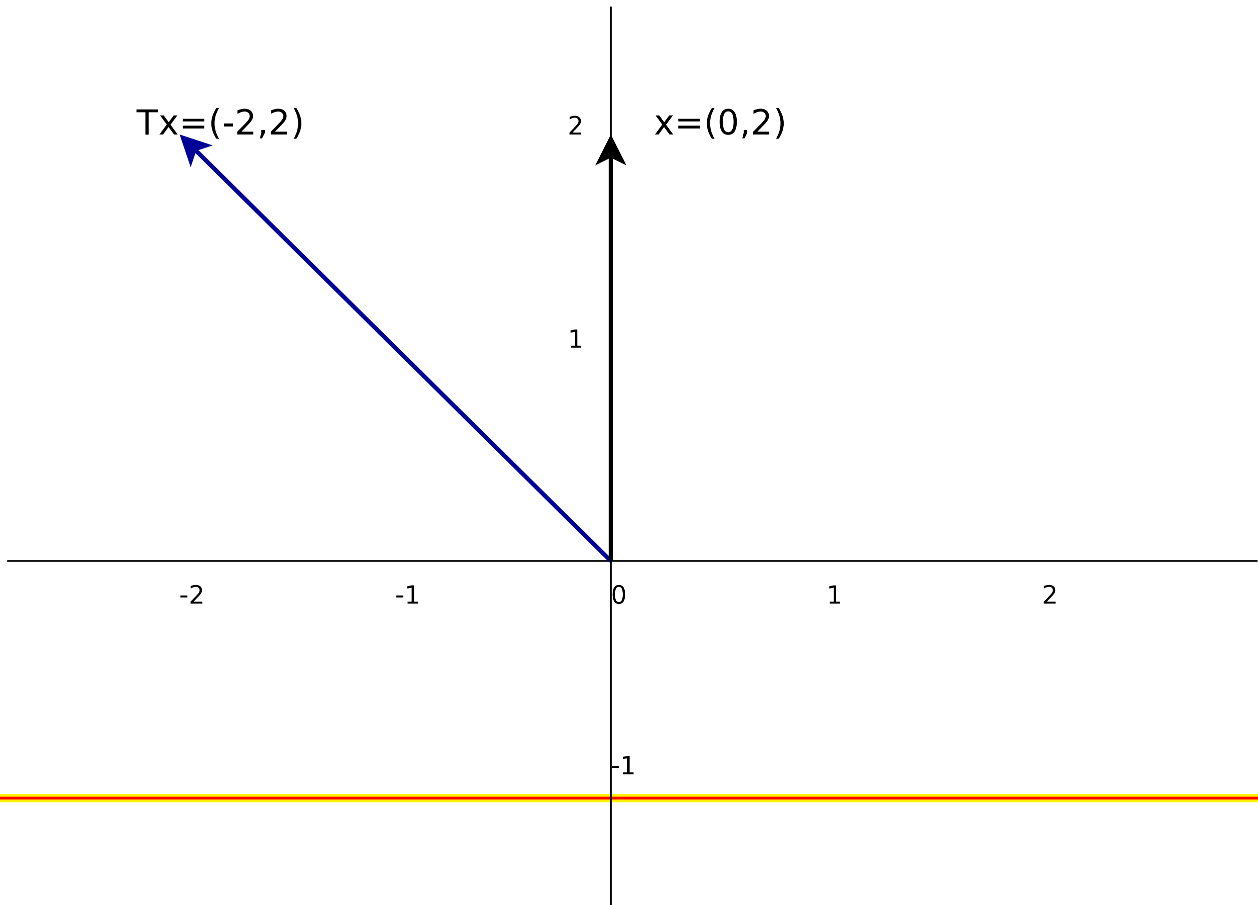
$$\begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$$

In this case, $k = -1$.

Horizontal Shears

Let's examine the action of T on some vectors in $\mathbb{R}^2$:

$$\text{let } \vec{x} = \begin{bmatrix} 0 \\ 2 \end{bmatrix} \text{ then } T(\vec{x}) = A\vec{x} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$



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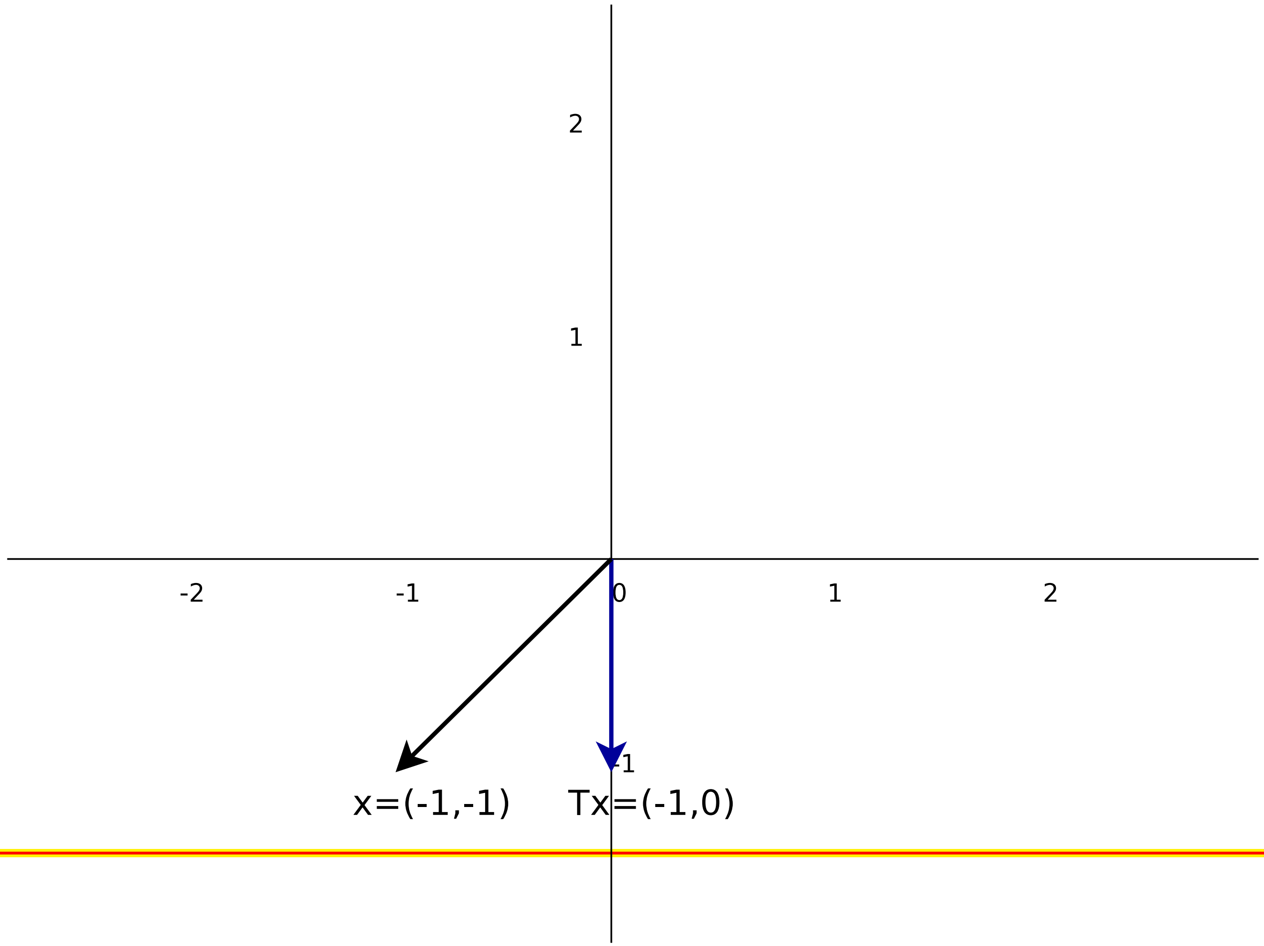
$$\text{let } \vec{x} = \begin{bmatrix} -1 \\ -1 \end{bmatrix} \text{ then } T(\vec{x}) = A\vec{x} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$



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