Scalings

Gene Quinn

Scalings

A **scaling** is a type of linear transformation

$$T : \mathbb{R}^n \to \mathbb{R}^n$$

that stretches or shrinks the vector it is applied to.

Scalings

A **scaling** is a type of linear transformation

$$T : \mathbb{R}^n \longrightarrow \mathbb{R}^n$$

that stretches or shrinks the vector it is applied to.

Like all linear transformations, a scaling is equivalent to multiplication by some matrix A:

$$T(\vec{v}) \;=\; A\vec{v}$$

for some matrix A.

Scalings

In the special case of two dimensions,

$$T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$$

the matrix A has the form

$$\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$$

where k is a **positive** constant.

Scalings

In the special case of two dimensions,

$$T : \mathbb{R}^2 \to \mathbb{R}^2$$

the matrix A has the form

$$\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$$

where k is a **positive** constant.

If $k > 1$, a scaling transformation is called a *dilation*

Scalings

In the special case of two dimensions,

$$T : \mathbb{R}^2 \to \mathbb{R}^2$$

the matrix A has the form

$$\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$$

where k is a **positive** constant.

If $k > 1$, a scaling transformation is called a *dilation*

If $k < 1$, a scaling transformation is called a *contraction*

Scalings

Let's consider the result of a scaling transformation on an arbitrary element $x \in \mathbb{R}^2$, (x_1, x_2):

$$T(\vec{x}) = A\vec{x} = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} kx_1 \\ kx_2 \end{bmatrix} = k \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = k\vec{x}$$

Scalings

Let's consider the result of a scaling transformation on an arbitrary element $x \in \mathbb{R}^2$, (x_1, x_2):

$$T(\vec{x}) = A\vec{x} = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} kx_1 \\ kx_2 \end{bmatrix} = k \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = k\vec{x}$$

Evidently the effect of this transformation is the same as multiplying the vector x by the scalar k.

Scalings

Suppose a linear transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is defined by:

$$T(\vec{x}) \;=\; A\vec{x}, \quad \text{for all } x \in \mathbb{R}^2$$

where

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

T qualifies as a scaling because its associated matrix A has the required form for a scaling:

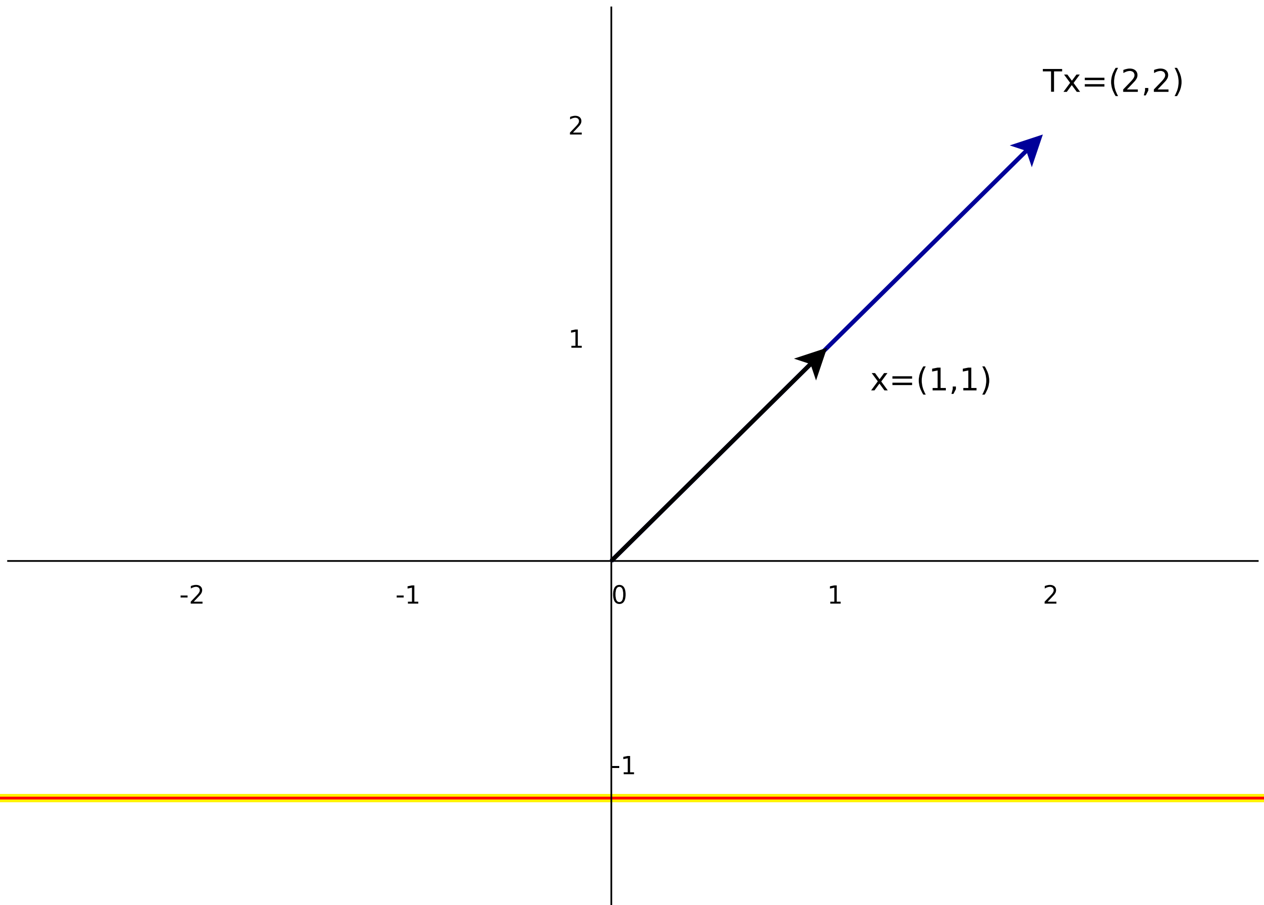
$$\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$$

In this case, $k = 2$.

Scalings

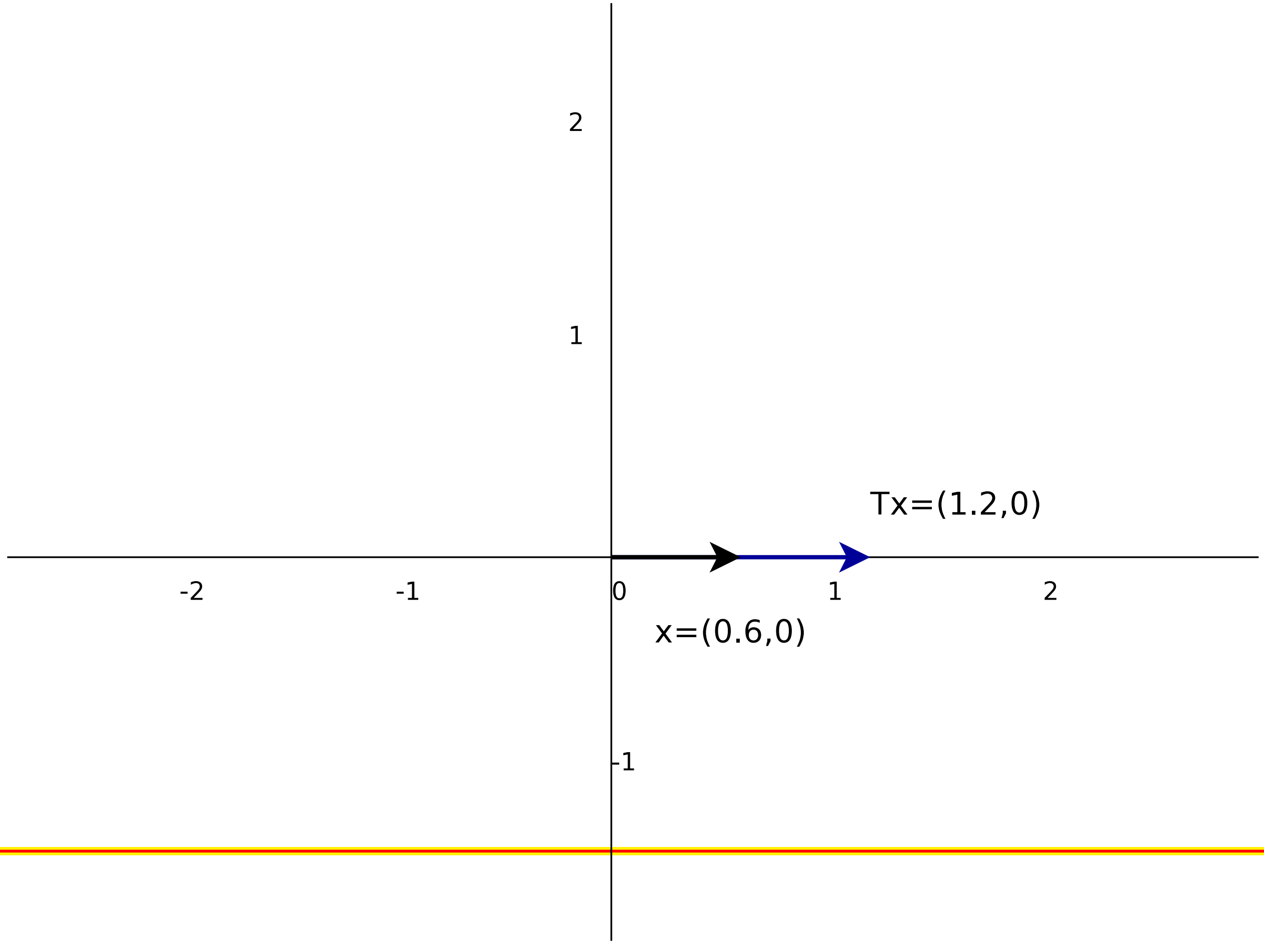
Let's examine the action of T on some vectors in $\mathbb{R}^2$:

$$\text{let } \vec{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \text{ then } T(\vec{x}) = A\vec{x} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$



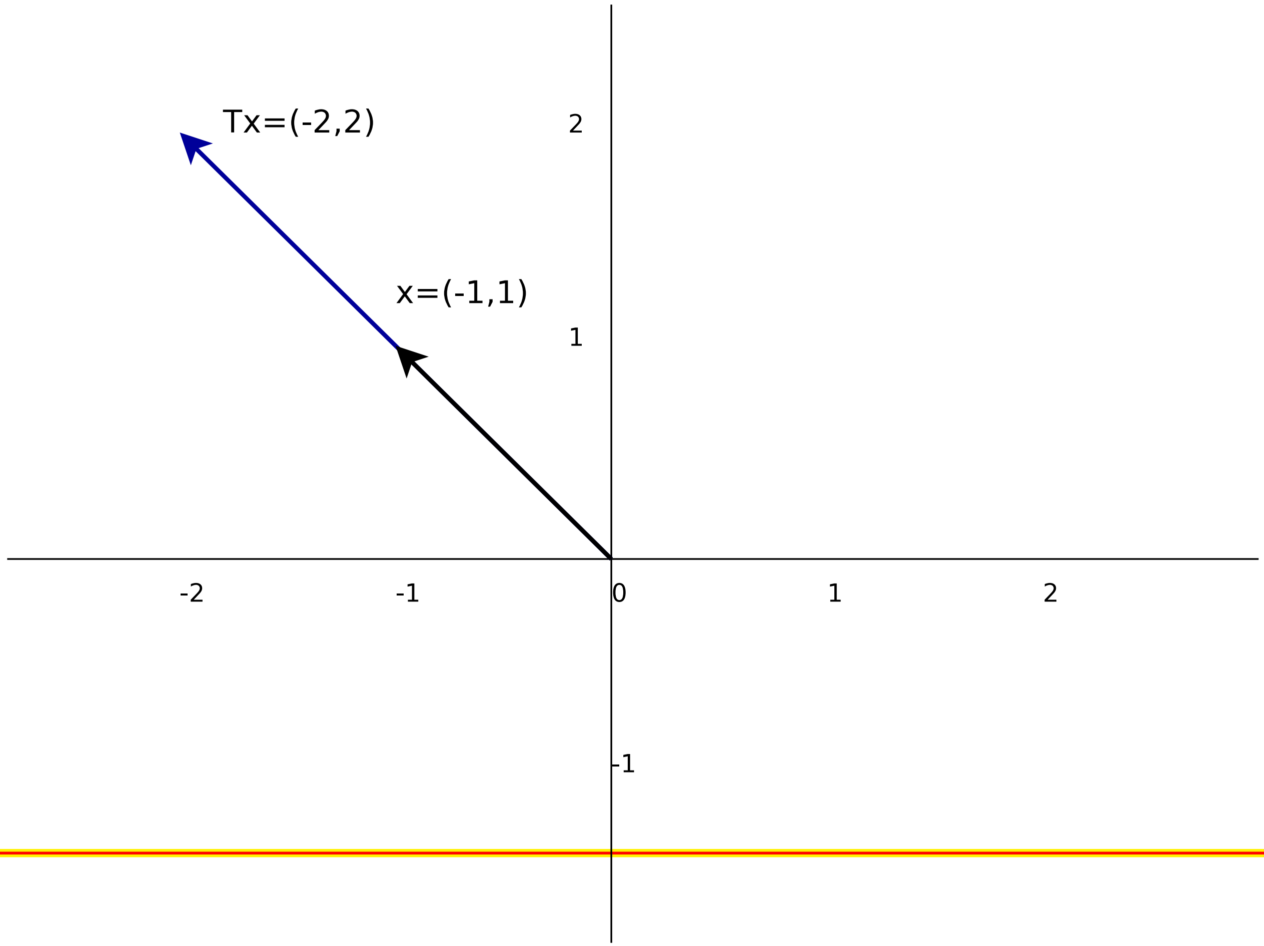
Scalings

let $\vec{x} = \begin{bmatrix} 0.6 \\ 0 \end{bmatrix}$ then $T(\vec{x}) = A\vec{x} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 0.6 \\ 0 \end{bmatrix} = \begin{bmatrix} 1.2 \\ 0 \end{bmatrix}$



Scalings

let $\vec{x} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ then $T(\vec{x}) = A\vec{x} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$



Scalings

Apparently the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$

$$T(\vec{x}) \;=\; A\vec{x}, \quad \text{for all } x \in \mathbb{R}^2$$

with

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

doubles the length of $\vec{x}$ without changing its direction.

We would say in this case that T is a *dilation*

Scalings

This time suppose a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ is defined by:

$$T(\vec{x}) \;=\; A\vec{x}, \quad \text{for all } x \in \mathbb{R}^2$$

where

$$A = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$$

T qualifies as a scaling because its associated matrix A has the required form for a scaling:

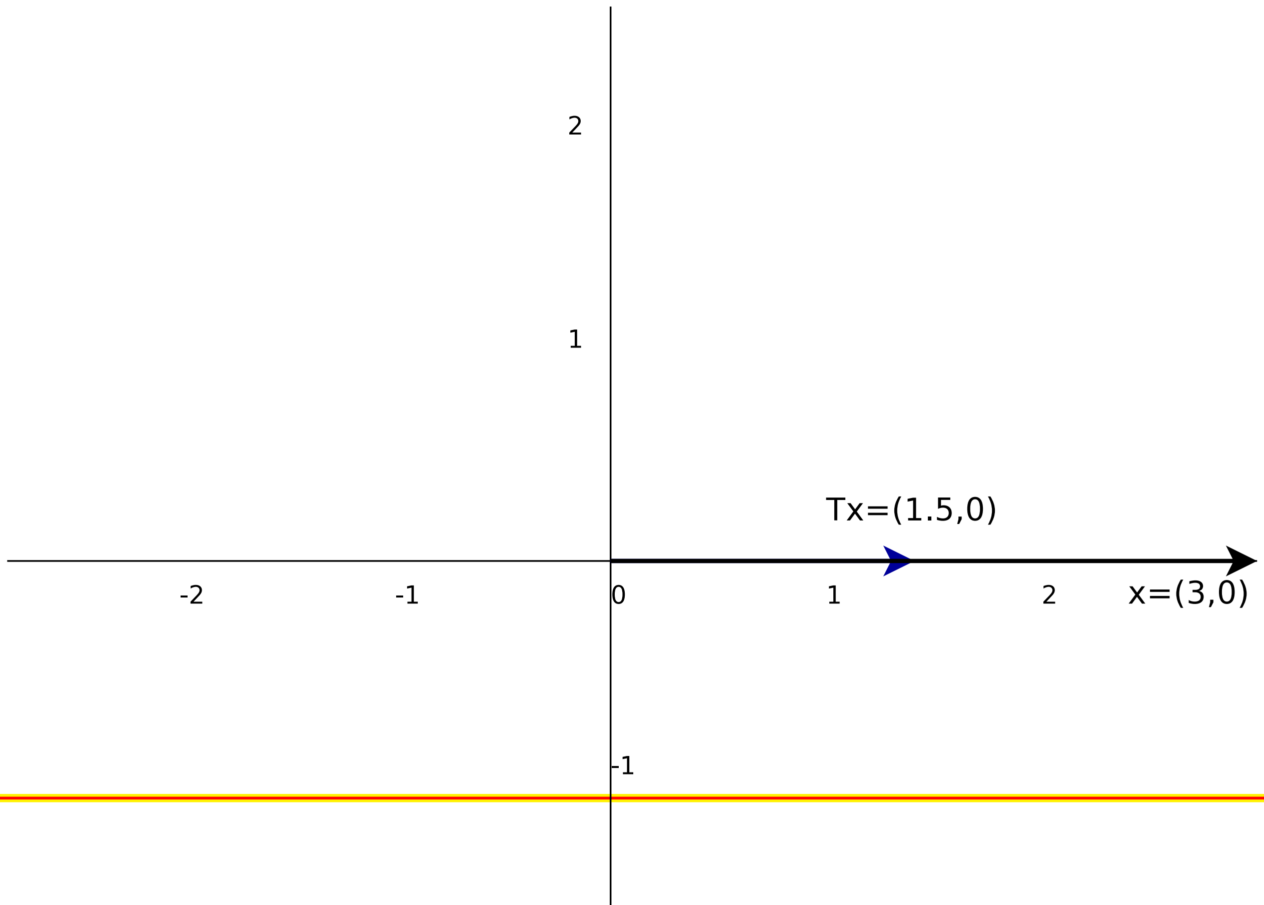
$$\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$$

In this case, $k = 0.5$.

Scalings

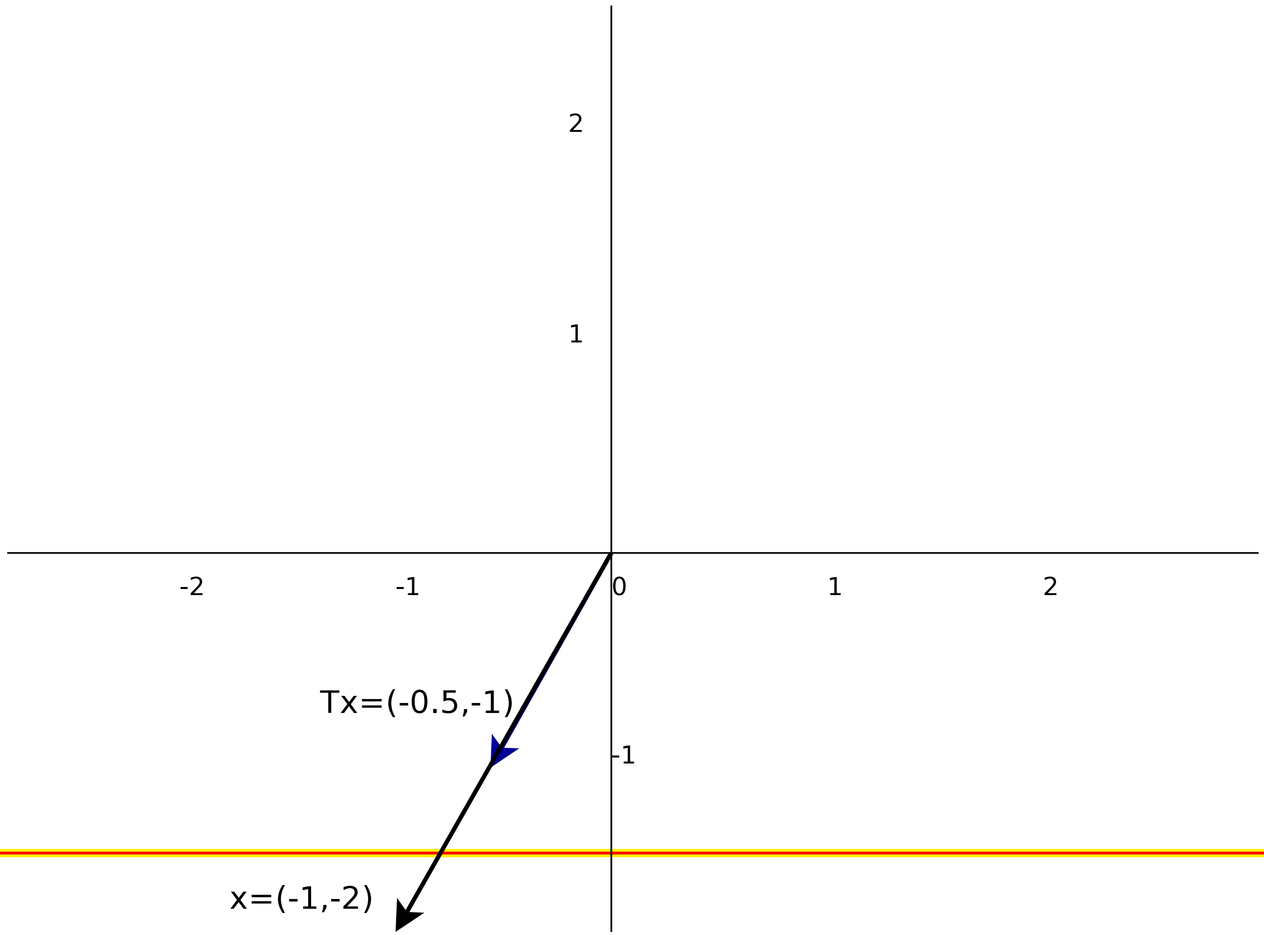
Let's examine the action of T on some vectors in $\mathbb{R}^2$:

$$\text{let } \vec{x} = \begin{bmatrix} 3 \\ 3 \end{bmatrix} \text{ then } T(\vec{x}) = A\vec{x} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} \begin{bmatrix} 3 \\ 3 \end{bmatrix} = \begin{bmatrix} 1.5 \\ 1.5 \end{bmatrix}$$



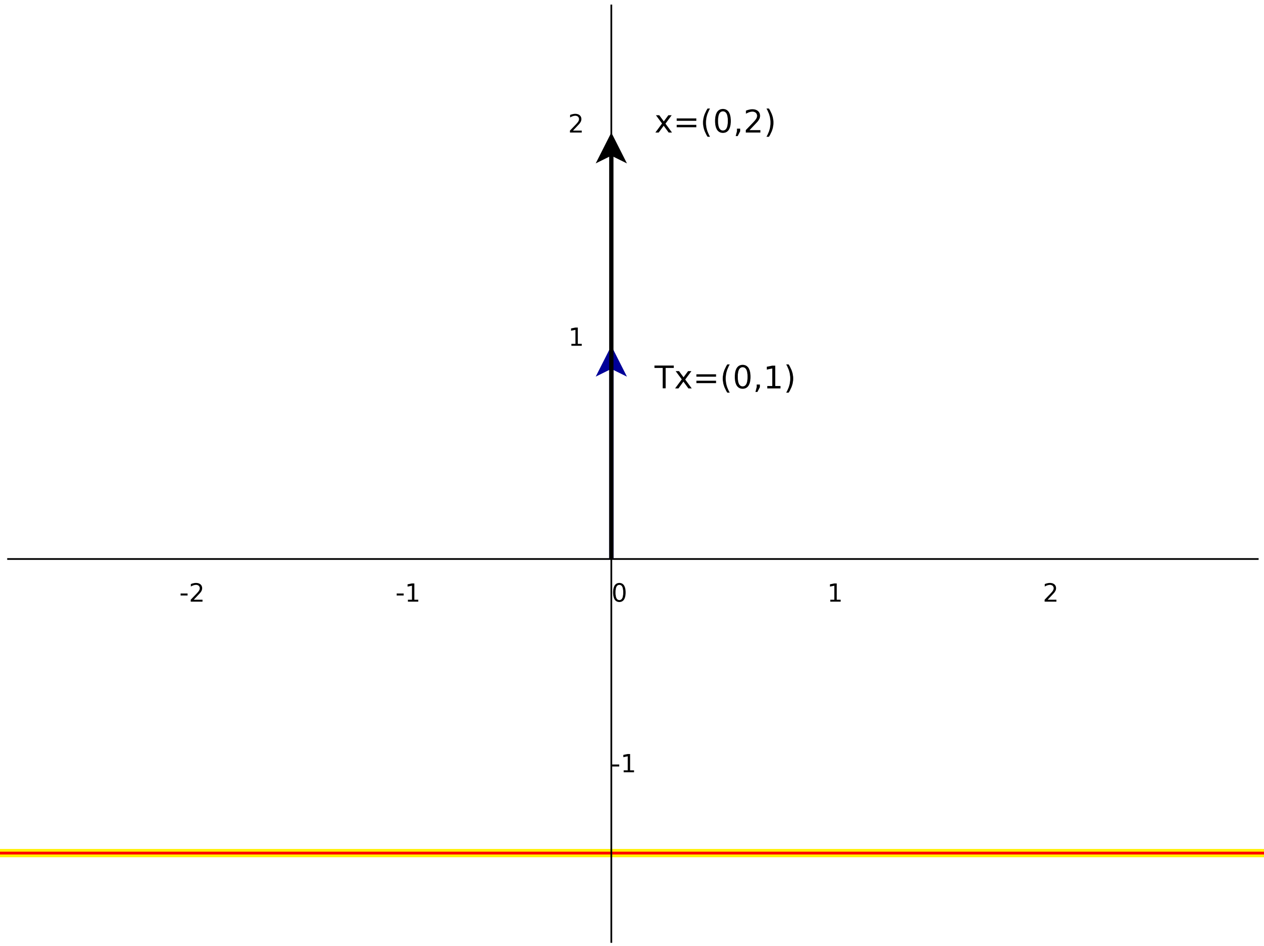
Scalings

let $\vec{x} = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$ then $T(\vec{x}) = A\vec{x} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} \begin{bmatrix} -1 \\ -2 \end{bmatrix} = \begin{bmatrix} -0.5 \\ -1 \end{bmatrix}$



Scalings

let $\vec{x} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$ then $T(\vec{x}) = A\vec{x} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$



Scalings

Apparently the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$

$$T(\vec{x}) \ = \ A\vec{x}, \quad \text{for all } x \in \mathbb{R}^2$$

with

$$A = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$$

cuts the length of $\vec{x}$ in half without changing its direction.

We would say in this case that T is a *contraction*