Relations

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An equation of the form

$$c_1\vec{v}_1 + \cdots + c_m\vec{v}_m = \vec{0}$$

is called a **relation** (or *linear relation*) amont the vectors $\vec{v}_1 \cdots \vec{v}_m$.

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For a given set V of vectors, nontrivial relations may or may not exist.

Relations

A set of vectors

$$V = \{\vec{v}_1, \ldots, \vec{v}_m\}$$

is linearly dependent if and only if there exists a nontrivial relation among them.

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Proof: Suppose a nontrivial relation exists among the vectors

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That is, there exist scalars $c_1, \ldots, c_m$ such that

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Let i be the highest index for which $c_i \neq 0$. Solving for $\vec{v}_i$ gives

$$\vec{v}_i = -\frac{c_1}{c_i}\vec{v}_1 - \cdots - \frac{c_m}{c_i}\vec{v}_m$$

which shows that $\vec{v}_i$ is redundant, so the members of V are linearly dependent.

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Now suppose teh vectors in V are linearly dependent. Then there is a redundant vector, say v_i, that can be written as a linear combination of the other elements of V:

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$$\vec{v}_i \; = \; c_1\vec{v}_1 + \cdots + c_m\vec{v}_m$$

In this case a nontrivial relation can be obtained by subtracting $\vec{v}_i$ from both sides:

$$c_1\vec{v}_1 + \cdots - \vec{v}_i + \cdots + c_m\vec{v}_m \; = \; 0$$

Relations and Kernels

If A is an $n \times m$ matrix associated with the linear transformation $T : \mathbb{R}^m \rightarrow \mathbb{R}^n$, and $\vec{x}$ is an element of $ker(A)$, then

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So, nonzero elements of $ker(A)$ correspond to nontrivial relations among the columns of A.