Name:

1) A continuous probability distribution is defined by:

$$f(x) = k \cdot (4 - x^2) \quad -2 \le x \le 2$$

a) Determine the value of k that makes f a valid probability distribution

b) If X is a random variable having this density function, find $E(X)$.

c) Find the variance of X.

d) Find the cumulative distribution function $F(x)$.

e) Find probability that $X < 1$.

2) IQ tests are scaled so that individual scores have a normal distribution with a mean of 100 and an standard deviation of 15.

a) What is the probability that a randomly chosen individual will score less than 128 on the IQ test?

b) What is the probability that a randomly chosen individual will score more than 95 on the IQ test?

c) Find the probability that a randomly chosen individual will score between 90 and 110 on the test.

d) Find the probability that a randomly chosen individual will score less than 85 or more than 115 on the test.

e) Find the 98th percentile of the distribution of IQ scores.

3) A random sample of size $n = 50$ is chosen from a population with mean $\mu = 78$ and variance $\sigma = 30$.

a) Find the expected value of the *sample mean* $\overline{X}$,

$$\overline{X} = \frac{1}{n} \sum_{i=1}^{50} x_i$$

b) What is the variance of $\overline{X}$?

c) Find the approximate probability that the mean will be between 80 and 85.

d) Find the approximate probability that a randomly chosen individual will score less than 75 or more than 81.

e) Find the approximate 75th percentile of the distribution of $\overline{X}$.

4) A random sample of size $n = 150$ is chosen from a population with finite mean and variance. The sample mean is $\overline{X} = 15.3$ and the sample standard deviation is $s = 3.1$.

a) Find an approximate 95% confidence interval for the population mean μ.

b) Find an approximate 99% confidence interval for the population mean μ.

5) A random sample of size $n = 20$ is chosen from a normal population. The sample mean is $\overline{X} = 175.3$ and the sample standard deviation is $s = 12.1$.

a) Find a 95% confidence interval for the population mean μ.

b) Find a 99% confidence interval for the population mean μ.

6) SAT scores are standardized to have a normal distribution with mean $\mu = 500$ and standard deviation $\sigma = 100$. Suppose a sample of 200 students from a large school district has a mean of 491.3. Test the hypothesis that the population mean for the school district is 500 against the alternative that it is less than 500 at the $\alpha = .05$ level.

7) An exit poll surveys 800 voters and finds that 483 voted for candidate A. The network will call the election in favor of the other candidate B if the null hypothesis H_0 that the proportion of voters favoring candidate A is 0.5 can be rejected at the $\alpha = 0.01$ level against the alternative hypothesis that the proportion is less than 0.5. Should the network call the election in favor of candidate B based on the exit poll?

8) A random sample of size $n = 25$ is chosen from a production run of gears. The mean gear diameter is $\overline{X} = 181.4mm$ and the sample standard deviation is $s = 0.4mm$. If the specification calls for a diameter of $180mm$, test the hypothesis that the mean diameter for the run is $180mm$ against the alternative that it is not equal to $180mm$ at $\alpha = 0.01$.