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Now let B represent the event "a black card drawn".

B is a subset of $\mathcal{S}$ with 26 elements, the black cards.

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In the card experiment, events A (queen) and B (black card) are *not* disjoint, because there are cards that belong to both events: the two black queens.

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This means we have to use the more general formula on page 55,

$$P(A \cup B) \;=\; P(A) + P(B) - P(A \cap B)$$

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The probability of the event $A \cup B$, which would be described as "A black card or a queen is drawn"

is expressed in terms of the probabilities of three events:

A	A queen is drawn	4 outcomes
B	A black card is drawn	26 outcomes
$A \cap B$	A black queen is drawn	2 outcomes

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Based on this assumption, we can assume that the probability of any event is simply $1/52$ times the number of outcomes contained in the event:

$$P(E) = \frac{N(E)}{52}$$

Using this principle, we can compute the probabilities of our three events

A	queen	4 outcomes	$P(A) = 4/52$
B	black card	26 outcomes	$P(B) = 26/52$
$A \cap B$	black queen	2 outcomes	$P(A \cap B) = 2/52$

Probability of Unions

If we think about the ways the outcome "A queen or a black card" can occur, we get:

One of the black cards	26 cards
One of the red queens	2 cards
	28 cards

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Since there are 28 cards contained in the event "A black card or a queen", the probability should be:

$$P(A \cup B) = \frac{28}{52}$$

Probability of Unions

Now returning to

$$P(A \cup B) \ = \ P(A) + P(B) - P(A \cap B)$$

we can substitute the probabilities for the right hand side:

$$P(A) = \frac{4}{52} \quad P(B) = \frac{26}{52} \quad P(A \cap B) = \frac{2}{52}$$

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This is the same answer we arrived at by counting the number of outcomes in the event.